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ARITHMETIC TESTS AND STUDIES IN THE PSYCHOLOGY OF ARITHMETIC

By

GEORGE S. COUNTS

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TABLE OF CONTENTS

CHAPTER		PAGE
I.	INTRODUCTORY STATEMENT	1
II.	THE NATURE OF THE TEST AND COLLECTION OF DATA	4
	The Test	
	Series A and B of the Courtis Tests	
	The Test Described	
	Addition	
	Subtraction	
	Multiplication	
	Division	
	Fractions	
	Time Allowance	
	Collection of Data	
	The Cleveland Test	
	The Grand Rapids Test	
	Summary Statement	
III.	GENERAL RESULTS	21
	Determination of Standards	
	Derivation of a System of Weights	
	The Use of the Test	
	Distributions	
	Accuracy	
	Summary	
IV.	TYPES OF ERRORS	53
	Addition	
	Subtraction	
	Multiplication	
	Division	
	Fractions	
	Fractions of Like Denominators	
	Fractions of Unlike Denominators	
	Summary	

CHAPTER PAGE

V. A Comparison of the Arithmetical Abilities of Certain Age and Promotion Groups 78

Age Groups

Method

Results

Promotion Groups

Methods

Results

Summary

VI. A Comparison of the Arithmetical Abilities of Certain Race Groups 112

Method

Results

Conclusions

VII. Summary and Conclusions 121

The Nature of the Test and Collection of Data

General Results

Types of Errors

A Comparison of the Arithmetical Abilities of Certain Age and Promotion Groups

A Comparison of the Arithmetical Abilities of Certain Race Groups

CHAPTER I

INTRODUCTORY STATEMENT

This investigation is a study of the arithmetical abilities or attainments of school children as measured by an arithmetic test. The study naturally falls into two divisions, the first including chapters ii, iii, and iv, the second, chapters v and vi. In the former, the test used in the investigation is described, and results are discussed which throw light on its use. In the latter, two special studies are made in which the test is used as a measuring instrument. These five chapters will now be described in greater detail.

In chapter ii it is shown that there is a need for a spiral test in the "fundamentals" of arithmetic to be used in diagnosing city, school, class, and individual weaknesses in the various operations included in the term "fundamentals." It is further pointed out that Series A and B of the Courtis standard tests are inadequate to meet this need. The test then, as developed, composed of 15 sets of different types of examples, is described and analyzed. This is followed by a statement concerning the collection of the data upon which the remainder of the study is based.

The purpose of chapter iii is fivefold: (1) In order that the test may be of the greatest value educationally it is necessary that standard attainments for children in the various grades in each of the 15 sets be determined. This is done on the basis of results from Cleveland and Grand Rapids. The validity of these results is discussed from the standpoint of the Courtis standard scores. (2) A system of weights is derived by which it is made possible to convert the scores made by a particular group or individual in the 15 different types of arithmetical operations into a single score to represent general arithmetical attainments of the individual or group. (3) The use of the test is discussed in detail, the method by which it may be employed to diagnose city, school, class, and individual weaknesses being shown. (4) Distributions of the scores

made by groups of children in the typical operations are discussed for the purpose of indicating the different types of individual reaction to examples of varying degrees of complexity and for the purpose of pointing out certain differences in the responses made to the "fundamentals" and to fractions. (5) The degree of accuracy with which the various types of examples are worked is shown, accompanied by a comparison of the curve of accuracy and the curve of "rights" for one of the sets.

Chapter iv is a study of errors, in which the types of errors made by children in working the different kinds of examples are analyzed. It is of value to the teacher to know what sorts of errors she may expect from the pupil when the latter encounters the different arithmetical operations. The frequency of these errors is also determined in order that the teacher may be able to apply the proper amount of emphasis at the various points of difficulty. Because of inability to isolate kinds of errors made in connection with some types of examples, since the study was confined to an examination of records made by pupils, this study is incomplete. It is necessary that it be supplemented by experimental data.

The problem presented by the study in chapter v is, in the first place, the problem of measuring the attainments of various groups of children for the purpose of discovering differences in four age groups throughout Grades 3–8 inclusive. In the second place, a study is made of certain promotion groups for the purpose of discovering differences. This division of the study has three parts: the first relates to the fast and slow pupils and is confined to the records of pupils in Grade 8-2; the second is concerned with a group of pupils repeating because of failure to do the work of the grade, a group repeating because of sickness, transfer of school, or similar cause, and a group of pupils making normal progress, the data for this study being secured from pupils in Grade 7-2 only; the third has to do with a group of pupils in Grade 8-2 who had failed below the sixth and another group who had failed above the fifth grade. The differences found are analyzed and interpreted.

In chapter vi a problem of the same general type as that of the previous chapter is encountered. The problem here is to deter-

mine whether or not there are differences in arithmetical attainments which follow racial lines. Owing to the meagerness of the data, this study is confined to five races, or nationalities, Americans, Hollanders, Germans, Swedes, and Slavs.

Owing to the fact that this entire study has been made on the basis of records made by pupils, it is in many particulars incomplete and tentative, for there are many matters that cannot be determined by an examination of records. Furthermore, the conditions under which the records were made were not sufficiently under control. It is therefore evident that it is necessary to supplement this study by experimentation.

CHAPTER II

THE NATURE OF THE TEST AND COLLECTION OF DATA

THE TEST

In connection with the Cleveland Survey the demand arose for an arithmetic test to measure the presence and absence of arithmetical attainments in the school children of that city. The sort of test desired was one that would, on the one hand, show the general standing of the city as a whole in the "fundamentals" of arithmetic and would, on the other hand, be diagnostic in its character, indicating school, class, and individual weaknesses in each of the different types of operations which enter into the solving of the more complex examples in each of the four fundamental operations.

SERIES A AND B OF THE COURTIS TESTS

It was felt by those in charge of the survey that no test had as yet been devised which would exactly fit their needs. Series A of the Courtis tests was unsatisfactory because, as Mr. Courtis himself has said, "the standards derived from the use of Series A . . . . are either complex or of questionable value, owing to the uncertainty of their meaning."[1] Tests Nos. 1, 2, 3, and 4 of this series are merely tests of knowledge of the tables in the four fundamental operations, and, since a pupil may know his tables perfectly and yet be quite unable to solve any of the more complex examples, and vice versa, these tests by themselves are of little value. Test 8, the only other test of the fundamentals in this series, is of doubtful value. In the first place, the form in which the examples appear is not the form to which the child is accustomed. For example, when called upon to add two or more numbers, the pupil does not ordinarily have them presented to him in this form, 30+735+123= . In order to work the example he must copy the three

[1] S. A. Courtis, *Manual of Instruction for Giving and Scoring the Courtis Standard Tests*, p. 7.

numbers in column form. This consequently makes necessary the copying of figures in the test, or else the performing of the operation in a wholly unaccustomed manner. In the second place, the use of the symbols introduces another factor. A pupil might be able to perform the required mathematical operation perfectly, yet fail on an example in this test because of unfamiliarity with the symbols. If it is desired to test the knowledge of symbols, a separate test should be devised for that purpose. In the third place, a particular score in this test may mean almost anything because of the complex nature of the test. For example, what may a score of "four" mean? It may mean either strength or weakness in any one of the four operations, or it may mean anything between these extremes.

Thus, since Series A is found to be quite unsatisfactory, let us turn to Series B of the Courtis tests. The latter, when used as a supplement to the former, or rather when substituted for Test 8 of that series, represents a distinct improvement over the earlier tests. The four tests in Series B are composed of four sets of complex examples in the four operations. Test 1 involves the addition of columns of 9 three-place numbers, Test 2 the subtraction of eight-place numbers from eight- and nine-place numbers, Test 3 the multiplication of four- by two-place numbers, and Test 4 the division of four- and five- by two-place numbers.

Series B supplemented by Series A is very good so far as it goes, but it does not go far enough. It makes possible a measure of the general attainment in each of the fundamental operations, but does nothing more. In a word, it is not diagnostic. For instance, suppose we have a pupil who knows his addition tables perfectly, as indicated by a record made in Test 1 of Series A, but fails miserably on Test 1 of Series B. These two facts about the pupil are worth knowing, but are of comparatively little value unless supplemented by other facts. Why he fails on the second test is not known. It may be because of failure to bridge the attention spans, or of inability to "carry," but the test throws no light on the question. It is just at this point that Series A and B of the Courtis tests break down. It is necessary to introduce, between the very simple type of example in the first series and the highly complex

type in the second series, tests representing types of intermediate complexity. This is, in fact, the logical evolution of Mr. Courtis' own system and is actually embodied in principle in his *Standard Practice Tests.*

THE TEST DESCRIBED

Since no existing test quite met the needs of the members of the survey staff, they took upon themselves the task of devising one. In this work the co-operation of Mr. Courtis was secured, with the result that to him is due whatever merit the test, as it now stands, may possess.

In order that the reader may get a clear impression as to the nature of the test, and that the discussion may be the more easily followed, the test is here reproduced in full. Passing over for the moment the first page of the test folder, since it does not constitute a part of the test, the test is seen to be composed of 15 sets, designated as Sets A, B, ... O.

An examination of the test shows it to be composed of four sets in addition (A, E, J, M), two in subtraction (B, F), three in multiplication (C, G, L), four in division (D, I, K, N), and two in fractions (H, O). Since the pupil begins with Set A and takes each set in its proper order, the spiral character of the test is apparent, a feature which deserves some further comment. The several sets in each operation are arranged in the test in the order of their complexity, but with them are interwoven the sets of the other operations. Thus a pupil first works on a set of examples in addition, then passes successively to sets in subtraction, multiplication, and division before encountering addition again. This changing from one type of operation to another lessens the strain on the pupil which is involved in a prolonged test of this sort.

ADDITION

As indicated above, there are 4 sets in addition. Set A involves the addition of the simple combinations, Set E the addition of columns of 5 one-place numbers, Set J the addition of columns of 13 one-place numbers, and Set M the addition of columns of 5 four-place numbers. The 65 examples of Set A were taken from Test 1, Series A, of the Courtis tests; the 16 examples of Set E,

ARITHMETIC EXERCISES

Name________________________________ Age today__________
Years Months

Grade__________ School_________________________________

Teacher_________________________ Date today____________

Have you ever repeated the arithmetic of a grade because of non-promotion or transfer from other school. If so, name grade_________

Explain cause____

Inside this folder are examples which you are to work out when the teacher tells you to begin. Work rapidly and accurately. There are more problems in each set than you can work out in the time that will be allowed. Answers do not count if they are wrong.

Begin and stop promptly at signals from the teacher.

	A	B	C	D	E	F	G	H
A								
R								

	I	J	K	L	M	N	O	
A								
R								

SET A—Addition—

													Ats.	Rts.
1	6	9	0	4	1	7	9	3	2	1	3	6		
2	6	5	1	2	3	7	6	0	4	5	8	9		
0	3	8	9	7	8	2	1	4	8	0	2	3		
7	2	1	9	6	0	5	6	7	9	5	7	1		
4	7	0	3	1	2	5	6	7	5	8	6	9		
6	9	8	5	4	9	8	0	2	1	3	5	0		
4	2	9	7	4	5	7	4	8	0	3	9	2		
3	2	3	8	0	2	1	9	6	0	4	1	8		
5	0	6	2	4	5	1	6	3	7	9	0	4		
7	4	3	1	8	9	0	2	3	4	8	6	5		

SET B—Subtraction—

										Ats.	Rts.
9	7	11	8	12	1	9	13	4	12		
9	3	6	1	3	0	7	8	3	6		
8	11	12	5	10	6	11	15	10	12		
0	9	7	1	2	0	7	8	9	4		
2	7	13	3	10	1	6	15	4	8		
1	5	7	2	5	1	3	9	2	3		
4	10	13	10	9	5	8	17	6	11		
4	7	5	1	4	5	6	9	4	8		
5	12	15	5	16	7	8	16	9	11		
0	9	6	3	8	0	5	7	1	4		

Ats. | Rts.

SET C—Multiplication—

3	4	9	0	5	4	2	7	4	9
2	7	8	2	6	1	9	6	0	5
9	5	4	7	6	2	3	9	0	7
1	2	8	0	5	1	3	6	5	4
1	2	7	0	8	7	3	9	2	4
6	8	7	6	3	1	8	9	0	3
1	4	8	0	4	1	6	8	0	9
5	4	9	3	5	4	2	8	7	3
1	3	6	0	3	2	6	7	5	4
7	4	8	0	9	2	3	9	5	6

SET D—Division—

3)9	4)32	6)36	2)0	7)28	9)9	3)21
6)48	1)1	5)10	2)6	4)24	7)63	6)0
8)32	1)8	5)30	8)72	1)0	9)36	1)7
2)10	7)42	1)1	6)18	3)6	4)20	7)49
1)3	2)8	6)6	3)27	8)64	1)2	4)16
5)0	3)24	9)63	2)4	8)24	7)7	2)18
6)42	3)0	7)21	4)4	3)15	9)81	7)0

Ats.	Rts.

SET E—Addition

5	2	9	2	6	1	4	9
2	8	8	8	3	4	6	7
2	8	0	5	4	2	5	1
0	5	7	0	8	5	3	5
4	1	6	6	8	4	4	3

6	2	6	8	5	4	1	3
7	7	2	5	9	0	4	7
8	3	3	1	6	8	1	2
5	4	9	3	3	5	8	9
5	1	3	8	8	5	4	6

SET F—Subtraction

616	1248	1365	1092	716
456	709	618	472	344
1267	1335	707	816	1157
509	419	277	335	908
1355	908	519	1236	1344
616	258	324	908	818
1009	768	1269	615	854
269	295	772	527	286

SET G—Multiplication—

2345	9735	8642	6789	2345
2	5	9	2	6
9735	2468	6789	3579	2468
9	3	6	3	7
5432	9876	8642	3579	9876
4	8	5	7	4
5432	3689	2457	9863	7542
8	5	6	4	7

				Ats.	Rts.
SET H—Fractions—					
$\frac{3}{5}+\frac{1}{5}=$	$\frac{6}{9}-\frac{4}{9}=$	$\frac{4}{9}+\frac{1}{9}=$	$\frac{8}{9}-\frac{7}{9}=$		
$\frac{1}{9}+\frac{5}{9}=$	$\frac{3}{7}-\frac{1}{7}=$	$\frac{1}{7}+\frac{4}{7}=$	$\frac{6}{7}-\frac{2}{7}=$		
$\frac{2}{9}+\frac{4}{9}=$	$\frac{4}{5}-\frac{1}{5}=$	$\frac{5}{8}+\frac{1}{8}=$	$\frac{6}{9}-\frac{5}{9}=$		
$\frac{7}{9}+\frac{1}{9}=$	$\frac{5}{7}-\frac{2}{7}=$	$\frac{5}{9}+\frac{2}{9}=$	$\frac{8}{9}-\frac{1}{9}=$		
$\frac{1}{8}+\frac{3}{8}=$	$\frac{6}{8}-\frac{1}{8}=$	$\frac{2}{7}+\frac{1}{7}=$	$\frac{5}{9}-\frac{4}{9}=$		
$\frac{2}{9}+\frac{6}{9}=$	$\frac{4}{8}-\frac{3}{8}=$	$\frac{4}{7}+\frac{2}{7}=$	$\frac{7}{9}-\frac{5}{9}=$		
SET I—Division—					
$4\overline{)55424}$	$7\overline{)65982}$	$2\overline{)58748}$	$5\overline{)41780}$		
$9\overline{)98604}$	$6\overline{)57432}$	$3\overline{)82689}$	$6\overline{)83184}$		
$8\overline{)51496}$	$9\overline{)75933}$	$8\overline{)87856}$	$4\overline{)38968}$		

SET J—Addition

Ats.	Rts.

7	9	4	7	2	9	6	7	7	8	9	4	3	2
5	2	5	1	9	6	9	1	8	0	5	3	1	1
4	4	8	9	4	2	6	5	5	7	3	7	7	6
2	8	1	4	8	4	7	1	4	1	4	7	6	6
6	2	4	3	5	7	0	4	1	8	6	0	9	1
0	7	8	2	1	1	4	6	8	5	2	2	6	8
5	5	5	8	5	3	3	5	2	1	3	9	3	6
1	3	1	5	2	9	7	3	1	3	9	5	4	9
8	6	3	2	4	2	1	3	3	7	2	6	5	7
3	1	9	7	3	3	6	7	9	4	2	3	4	5
2	4	6	7	6	8	0	6	8	9	8	4	2	2
9	8	3	1	7	5	6	1	4	4	5	8	9	2
9	8	5	9	6	5	6	7	5	4	6	8	9	4

SET K—Division—

21)273	52)1768	41)779	22)462	31)837
42)966	23)483	72)1656	81)972	73)1679
21)294	62)1984	31)527	52)2184	41)984
32)384	51)2397	82)1968	71)3692	22)484
41)1681	33)693	61)1586	53)1166	31)496

SET L—Multiplication—

				Ats.	Rts.
8246 29	3597 73	5739 85	2648 46		
4268 37	7593 64	6428 58	8563 207		

SET M—Addition—

7493 9016 6487 7591 6166	8937 6345 2783 4883 1341	8625 4091 3844 8697 7314	2123 1679 5555 6331 6808	5142 0376 4955 9314 5507	3691 4526 7479 2087 8165
5226 2883 2584 0058 2398	9149 8467 0251 7535 5223	6268 7725 8331 5493 3918	9397 6158 3732 4641 7919	7337 2674 9669 5114 8154	8243 6429 9298 7404 2575

SET N—Division—

67)32763	48)28464	97)36084	59)29382
78)69888	88)34496	69)40296	38)26562

SET O—Fractions—

			Ats.	Rts.
$\frac{11}{15}+\frac{1}{6}=$	$\frac{9}{14}-\frac{1}{4}=$	$\frac{3}{5}\times\frac{5}{6}=$		
$\frac{5}{6}-\frac{2}{21}=$	$\frac{5}{6}\times\frac{19}{20}=$	$\frac{11}{12}\div\frac{5}{8}=$		
$\frac{1}{6}\times\frac{3}{10}=$	$\frac{5}{6}\div\frac{11}{15}=$	$\frac{5}{12}+\frac{2}{8}=$		
$\frac{20}{21}+\frac{1}{6}=$	$\frac{3}{4}+\frac{3}{18}=$	$\frac{3}{8}-\frac{3}{10}=$		

Instructions for Examiners

Have the children fill out the blanks at the top of the first page. Have them start and stop work together. Let there be an interval of half a minute between each set of examples. Take two days for the test; down through I the first day, and complete the test on the next day. The time allowances given below must be followed exactly.

Set A.........30 seconds	Set F.......... 1 minute	Set K........2 minutes
Set B.........30 seconds	Set G.......... 1 minute	Set L.........3 minutes
Set C.........30 seconds	Set H.........30 seconds	Set M........3 minutes
Set D.........30 seconds	Set I.......... 1 minute	Set N........3 minutes
Set E.........30 seconds	Set J.......... 2 minutes	Set O........3 minutes

Have the children exchange papers. Read the answers aloud and let the children mark each example that is correct, "C." For each set let them count the number of problems attempted and the number of C's and write the numbers in the appropriate columns at the right of the page.

The records should then be transcribed to the first page. Please verify the results set down by the pupils.

the 14 examples of Set J, and the 12 examples of Set M were taken wholly or in part from Lessons 4, 23, and 27, respectively, of the *Courtis Standard Practice Tests.*

These 4 types of examples in addition were chosen because the solution of the examples in each succeeding set involves a mental process not present in the immediately preceding set, which marks it off as a type. Thus Set A represents the very simplest sort of addition, the combining of 2 one-place numbers. In Set E the pupil must not only combine two numbers, but must hold this sum in his mind and combine it in turn with a third number, and so on through four combinations. At first glance Set J seems to be of the same type as Set E, the difference being merely one of quantity, but such is not the case. Twelve combinations must be made instead of four. Now the span of attention has limits. Anyone who has ever attempted to add a long column of figures knows what this means. The addition of one figure after another from the first figure in the column to the last is not one continuous process, but is broken up into segments. That is, the individual adds up to a certain point, holds the sum in his mind as the attention wavers, and then continues the addition of the column as the attention returns. This is called "bridging the attention spans" and is a mental process called forth in the addition of the long columns in Set J. There is one other operation that the pupil must learn to perform successfully before he can become a competent adder, and that is "carrying." For testing ability to perform this operation Set M appears in the test. In the addition of these columns the pupil must "carry" a result forward from the addition of one column to the next. Thus the 4 sets in addition indicate ability or lack of ability (1) in performing the simple addition combinations, (2) in adding a third number to a sum secured by the addition of two numbers, (3) in bridging the attention spans, and (4) in "carrying."

SUBTRACTION

There are but 2 sets in subtraction in the test. The first, Set B, is made up of the simple combinations; and the second, Set F, involves the subtraction of three-place numbers from three- and

four-place numbers. The examples in the former set were taken from Test 2, Series A, of the Courtis tests, and those in the latter from Lesson 20 of the *Courtis Standard Practice Tests.*

Subtraction is confined to 2 sets because, for diagnostic purposes, they are sufficient. The only operation that is added in the more complex forms of subtraction, not found in the simple combinations, is that of borrowing. This is demanded in the examples of Set F just as much as in the larger examples.

MULTIPLICATION

Multiplication appears in 3 sets. Set C involves the simple combinations, Set G the multiplication of four-place by one-place numbers, and Set L the multiplication of four-place by two-place numbers. The 50 examples in Set C were taken from Test 3, Series A, of the Courtis tests; the 20 examples in Set G were specially devised under the supervision of Mr. Courtis for this test; and the 8 examples in Set L were taken from Test 3, Series B, of the Courtis tests.

The first set tests knowledge of the tables. In the second the pupil must "carry" results forward. And in the third, Set L, the operation is further complicated by the demand for knowledge of the mechanics of handling the product of the multiplication and the second term of the multiplier. The addition of the partial products is also introduced.

DIVISION

Four sets are given over to division, D, I, K, and N. The simple combinations appear in Set D, the division of five-place by one-place numbers in Set I, the division of three- and four-place numbers by two-place numbers in Set K, and the division of five-place numbers by two-place numbers in Set N. The 49 examples in the first set were taken from Test 4, Series A, of the Courtis tests; the 12 examples in Set I were taken from Lesson 31 of the *Courtis Standard Practice Tests;* and the other two sets, K and N, made up of 25 and 8 examples, respectively, were specially devised for this test.

As in the sets for the other three operations, the attempt was here made to introduce into the test examples embodying the

different types of difficulty that are encountered in division. Set D tests knowledge of the tables. Set I is made up of more complex examples in short division which differ from the examples in Set D by the introduction of the operation of carrying. Sets K and N are sets in long division. The former represents the very simplest type of this operation, since there is no carrying required in the multiplication and no borrowing in the subtraction. The latter, on the other hand, is much more complex, involving both carrying and borrowing.

FRACTIONS

For the purpose of testing the ability of pupils to apply the four fundamental operations to the working of fractions 2 sets of fractions were placed in the test, Set H and Set O. Both sets, the one made up of 24 examples and the other of 12, were specially devised for this test.

The examples in Set H are very simple, involving the addition and subtraction of fractions of like denominators. In Set O fractions of unlike denominators are to be added, subtracted, multiplied, and divided. These sets of fractions, it will be noted, differ from the other sets in that they are not homogeneous. In the first there are two different types of operations to be performed, and in the second there are four. This is freely acknowledged as a defect. But, since the test was to be used in the survey, it was necessary that its scope be limited; and, since the testing of attainments in fractions was felt to be more or less experimental, it was thought that the fractions should be sacrificed rather than the fundamental operations.

TIME ALLOWANCE

A word should be said about the time allowances given to the several sets. The child is not allowed to begin with the first set and to work an indefinite time on it or any following set. On the contrary, as indicated by the time allowances given on the last page of the test, the pupil is allowed to work a specified time on each set. This time ranges from 30 seconds for the easier sets to 3 minutes for the more difficult sets. In each case the attempt was made to make the time allowance large enough to enable even

the slowest pupil to work at least one of the examples, and yet small enough to prevent even the most rapid pupil from exhausting the possibilities of the set. Thus the test is a speed test with a definite time allowance given to each of the 15 sets.

COLLECTION OF DATA

The data on which the present study is based were secured from two sources, viz., Cleveland and Grand Rapids. Since there were slight differences in the tests themselves and in the giving of the tests in the two cities, each will be treated separately.

THE CLEVELAND TEST

The test as given to the children of the Cleveland schools was slightly different from that just described. In the Cleveland test the result "21" was repeated so frequently in Set K that some of the pupils taking the test, after working several examples of the set and finding the answers to be "21" in almost every instance, wrote down "21" as the answer to the remaining examples without actually working them. In the light of this experience Set K was modified so as to avoid the repetition of this result. Set L was modified by giving more space for working the examples, because the Cleveland results showed that insufficient space had been given. Set O was also modified. In its earlier form the examples in the addition of fractions constituted one column, those in subtraction another, those in multiplication another, and those in division another. When they appeared in this form it was found that quite frequently a pupil would select the examples in multiplication and avoid the more difficult examples of the other operations. To place a check on this tendency the four types of examples were intermingled, as seen in the test in its present form.

The tests were given on June 4, 7, and 8, 1915, to the B sections of Grades 3–8 inclusive. The teachers gave the tests, following the instructions given on the test sheet and certain other instructions sent out to the principals of the schools from the office of the superintendent.[1] The scoring was done by the pupils under the supervision of the teacher.

[1] Charles H. Judd, *Measuring the Work of the Public Schools*, p. 245.

THE GRAND RAPIDS TEST

The test as described in this chapter was given to both sections of Grades 3–8 inclusive on February 28 and 29 and March 1, 2, and 3, 1916. A great deal more care was taken here than in Cleveland to insure the results against error. In the first place, the writer was present at a meeting of the principals from all of the schools, where the test was carefully gone over and the method of giving the test explained. In the second place, the request was made that one person, preferably the principal, do all the timing in each school, and that the testing be begun in the lower grades and proceed upward, so that the examiners might be somewhat experienced in the giving of the test when the more important grades were tested—more important because it is only in the upper three grades that the children are able to work examples in all the sets. In the third place, the teachers and the pupils in the Grand Rapids schools were familiar with the Courtis practice tests. The teachers were consequently to some degree experienced examiners, and the children were acquainted with the signals for beginning and stopping work. In the fourth place, the writer personally conducted the tests in 50 classes in 8 schools.

TABLE I

NUMBER OF CLASSES TESTED

Grade	Cleveland	Grand Rapids	Total
3	85	64	149
4	87	62	149
5	90	58	148
6	87	53	140
7	86	46	132
8	85	31	116
Total	520	314	834

From these two sources, as shown in Table I, results were secured from 834 classes, 520 in the Cleveland schools and 314 in the schools of Grand Rapids. The number of classes is given rather than the number of children tested because the medians in the general tables to be discussed in the following chapter are medians of class standings and not of individual standings.

SUMMARY STATEMENT

In summary, the present test is a speed test which measures attainments and indicates weaknesses in the four fundamental operations and fractions. In addition it tests knowledge of tables, the ability to add short columns, to bridge the attention spans, and to "carry"; in subtraction it tests knowledge of the tables and the ability to "borrow"; in multiplication it tests knowledge of the tables, ability to "carry," and ability to add in connection with multiplication; in division it tests knowledge of the tables, ability to "carry" in short division, and ability to solve two types of examples in long division, the one involving neither "carrying" nor "borrowing" and the other involving both; and it tests the ability to apply these four fundamental operations to the working of examples in fractions.

The test was given to, and results secured from, 834 classes in the schools of Cleveland and Grand Rapids. In both cities the test was given almost entirely by the teachers. In Cleveland the teachers were inexperienced in giving tests, while in Grand Rapids they were all more or less familiar with the Courtis tests.

CHAPTER III

GENERAL RESULTS

DETERMINATION OF STANDARDS

In order that the test may be of the greatest educational value it is necessary that standard scores be determined for the several grades in each of the sets. These scores must of course be determined empirically, that is, on the basis of what children actually do.

As indicated in the previous chapter, more or less valid results were secured from two large school systems, Cleveland and Grand Rapids. These results were tabulated, and measures of central tendency computed. The median was chosen for this measure for two reasons: (1) since it is not disproportionately affected by an extreme case, it in large measure eliminates errors due to overtiming or undertiming; (2) the median is easily computed. These two facts make the median a highly desirable average, especially when such an enormous body of material must be handled as is necessarily the case in the survey of a large school system.

The method used to secure a final average score for each of the sets of the tests was as follows: First, the median of each of the 90 Cleveland schools (more or less depending on the grade) in a particular grade was found; secondly, the median of these medians was computed to get an average for Cleveland as a whole; thirdly, the same thing was done for Grand Rapids; fourthly, the medians of the two cities were averaged to get tentative standard scores for the different sets of the test. The test was given to the B sections only in Cleveland and to both sections in Grand Rapids, but since it was given in Cleveland at the close of the term (June) and in Grand Rapids at the beginning of the term (February, March), in order to get the standard score the results from the lower sections in Cleveland were averaged with results from upper sections in Grand Rapids.

These standard scores found by averaging the Cleveland and Grand Rapids medians appear in Table II. An examination of the

table shows that the average scores made in Set A, simple addition, by third-grade pupils in the time allowance (30 seconds) was 13.4 examples; by fourth-grade pupils, 17.1 examples, etc. The absence of a score, as in the earlier grades for Sets H, K, L, N, and O, indicates that the pupils in that grade were unfamiliar with the type of operation demanded.

TABLE II

AVERAGES OF MEDIAN SCORES IN EACH ARITHMETIC TEST FOR GRADES 3–8. CLEVELAND AND GRAND RAPIDS

Set	Grade					
	3	4	5	6	7	8
A	13.4	17.1	21.9	24.9	27.0	28.9
B	8.9	12.8	16.6	19.5	21.1	25.8
C	6.5	11.7	14.8	16.8	18.2	19.9
D	6.3	11.4	15.0	17.7	20.3	22.8
E	4.3	5.0	5.9	6.7	7.4	8.0
F	2.0	4.5	6.6	7.7	9.1	10.6
G	2.0	3.6	5.1	5.5	6.0	6.7
H	....	....	5.6	6.0	7.7	8.6
I	0.6	1.0	1.7	3.1	4.0	4.7
J	1.9	3.0	3.9	4.4	5.1	6.1
K	....	4.0	5.6	7.0	9.4	11.4
L	....	1.7	2.7	3.2	3.8	4.4
M	1.4	2.4	3.4	4.1	4.7	5.4
N	....	0.8	1.1	1.6	1.9	2.4
O	....	....	....	3.3	4.3	5.2

It is freely conceded by the writer that, because of the complexity of the test and the difficulties encountered in following the time allowances, and because of the fact that the test was quite largely given by persons with little or no training in testing, it is very likely that many errors were made in the giving of the test. Now the important question that arises is the nature of the errors made. If they were of a compensating sort—that is, if it were purely a matter of chance whether the examiner overtimed or undertimed—the errors made in one direction were offset by those made in the other. If, on the other hand, the errors were of the cumulative type—that is, if for any reason the examiners tended to overtime more than undertime, or vice versa—the errors would

not offset one another, and an error would enter into the final results. On first thought it would seem that, since the tests were being given in connection with a survey to determine the standing of a city in arithmetical attainments, as well as the relative standings of the individual schools within the city, there would be a tendency for the teachers to overtime rather than to undertime.

Some light may be thrown on our problem if we turn to Table III. In the description of the test it was said that five of the sets—A, B, C, D, and L—were taken over from Series A and B of the Courtis standard tests. It is therefore possible to make a comparison between the Cleveland–Grand Rapids average for each of these sets and the Courtis standard scores. This comparison is made in Table III and Diagram 1.

TABLE III

RESULTS OF CLEVELAND AND GRAND RAPIDS TESTS COMPARED WITH COURTIS STANDARDS

Grade	Score	Set A	Set B	Set C	Set D	Set L
3	Cleveland and Grand Rapids	13.4	8.9	6.5	6.3	......
	Courtis Standard	*13.0*	*9.5*	*8.0*	*8.0*	......
4	Cleveland and Grand Rapids	17.1	12.8	11.7	11.4	1.7
	Courtis Standard	*17.0*	*12.5*	*11.5*	*11.5*	*0.8*
5	Cleveland and Grand Rapids	21.9	16.6	14.8	15.0	2.7
	Courtis Standard	*21.0*	*15.5*	*15.0*	*15.0*	*2.0*
6	Cleveland and Grand Rapids	24.9	19.5	16.8	17.7	3.2
	Courtis Standard	*25.0*	*19.0*	*18.5*	*18.5*	*2.8*
7	Cleveland and Grand Rapids	27.0	21.1	18.2	20.3	3.8
	Courtis Standard	*29.0*	*22.0*	*20.5*	*22.0*	*3.3*
8	Cleveland and Grand Rapids	28.9	25.8	19.9	22.8	4.4
	Courtis Standard	*31.5*	*24.5*	*22.5*	*24.5*	*4.0*

The Courtis standards are supposed to represent June attainments, while the Cleveland–Grand Rapids average represents February or March attainments, almost a half-year behind the Courtis standard. A word should be said, however, about Set L. Very little reliance can be placed upon this comparison because the

Courtis standards in this case are purely tentative. For that reason no graphical representation is made of the comparison.

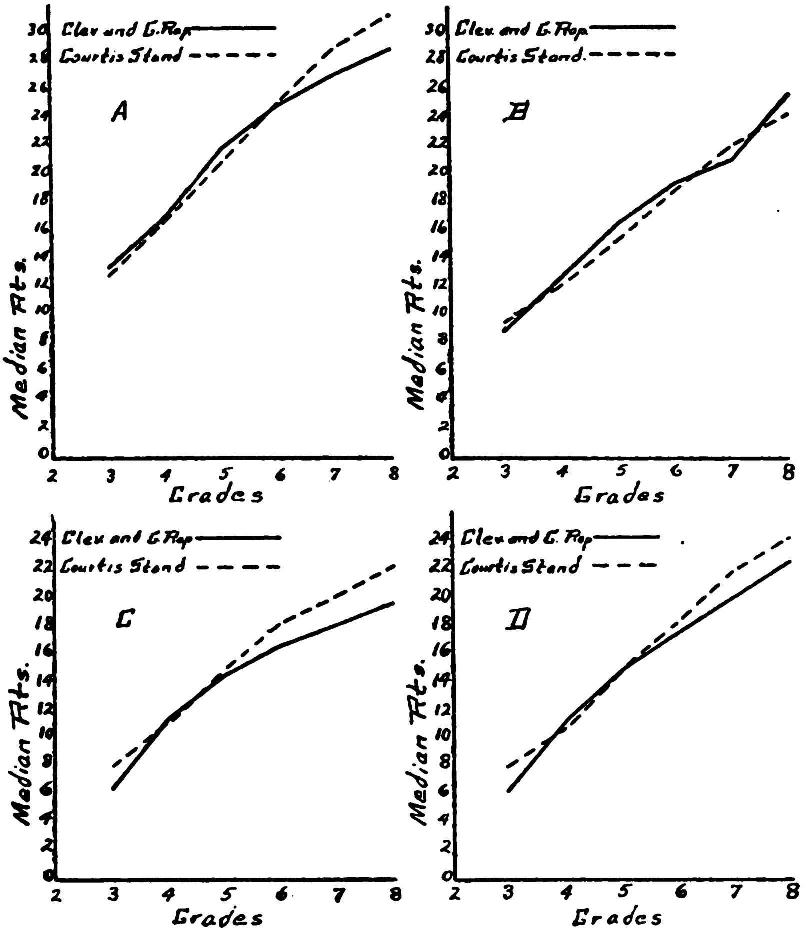


DIAGRAM I.—Results secured from Cleveland and Grand Rapids in Sets A, B, C, and D compared with the Courtis standards.

From the table and the diagram it is seen that, so far as the seventh- and eighth-grade attainments in the four sets of simple combinations are concerned, the Courtis scores are higher than the Cleveland–Grand Rapids scores. In the third grade the same

difference is noted, while in the intermediate grades the scores quite closely agree. Two facts deserve comment. In the first place, the differences on the whole favor the Courtis scores, and the differences are about as great as they should be in view of the fact that the Courtis scores represent an advantage of almost half a grade. The presumption is strong, therefore, that the errors which are very likely to have accompanied the giving of the test were of the compensating type in these four sets at least. In the second place, there seems to be a characteristic difference in the forms of the two curves of progress from grade to grade. The Courtis scores indicate uniform progress from grade to grade, while the Cleveland–Grand Rapids scores show, though not emphatically, to be sure, the progress to be less rapid with each successive grade. In other words, the latter tends to resemble in certain respects the typical learning curve. Thus it would seem that the question has not yet been answered whether, during progress through the grades, the limits which are ordinarily set to improvement through practice are completely offset by the maturing of the pupil. The Courtis results seem to indicate that these limits are offset, while the results from Cleveland and Grand Rapids seem to point to the contrary.

To return to the question of error in the final result, what may be said concerning the reliability of the scores for Sets E to O inclusive? Arguing from the Courtis standard scores, the scores for A, B, C, and D seem to be free from any considerable error. Through experience in giving the test the writer has come to the conclusion that accurate timing is more difficult in connection with these first sets than with the later sets, because of the short time allowances in the former. The same absolute error in two given cases is relatively a greater error where the time allowance is small than where it is large. It would at least seem that there is no reason for thinking that the later sets were not given as accurately as the first four. To this last statement an exception should possibly be made in the case of Set H. To this set an allowance of but 30 seconds is given, while a minute is given to each of the two preceding sets. There is undoubtedly a tendency for the examiner to allow a minute for this set also, so that there is probably a cause of error operating in Set H that is not present in the other sets.

From the foregoing it would seem that, arguing from the Courtis standards to Sets A, B, C, and D, and from these four sets to the remaining sets, with the possible exception of H, the average scores for the several sets made by the pupils in the lower sections in the Cleveland grades and the upper sections in the Grand Rapids grades constitute reliable standards for midyear attainments. These standards may therefore be tentatively accepted, subject of course to revision as returns are secured from other cities.

DERIVATION OF A SYSTEM OF WEIGHTS

It is desirable for certain purposes that some method be found of equating the scores made in the different sets by a particular system, school, class, or individual so that a single score, the summation of the scores made in the several sets, may be obtained to indicate general attainment in the "fundamentals." In order to do this, a unit must first be found in terms of which the score made in each of the sets may be stated.

In essence the equating of the sets resolves itself into a statement of their relative difficulties. There are two factors that constitute the criteria of difficulty: the first is speed, the second is accuracy. Since accuracy by itself means almost nothing, since the number of examples attempted likewise means but little, and since the number of examples worked correctly in a given time includes a measure of both speed and accuracy, it seems to the writer that the latter is as valid a gauge of difficulty as any that might be chosen. Having accepted this criterion, the equating of the sets is a very simple matter. A second's work might be taken as the unit. Then, if it required on the average three seconds to work one example and two seconds to work another, their relative difficulties would be as three is to two. For the sake of convenience, however, we may take the average time required to work an example in one of the sets as a unit. A value of 1.0 is then given to each of the examples worked in that set, with values for the examples of the other sets varying inversely as the speed with which they can be worked.

In the present study it is suggested that the average time required to work an example in Set A be accepted as the unit, and

that each example correctly worked in this set be therefore given a value of 1.0. An example of this set is chosen because of its size and stability. It is a smaller quantity than any other unit would be, because on the average a pupil works more examples of this type than of any other. It is more stable than any other, there being least variation from individual to individual in the records made in this set. A second suggestion is that the system of weights be derived from the records made by eighth-grade children. It is true that the relative difficulties of the sets are not the same from grade to grade. Set N is, for example, not only absolutely much more difficult for the fourth grade than for the eighth, but relatively much more difficult. Of course it is possible to make a system of weights for each grade, but that has its disadvantages. The thing desired is a system of weights that will show progress from grade to grade. If the system is changed with every grade, there is no intelligible relation between the score made by one grade and that made by the grade above or the grade below. Again, it would seem that the relative difficulties of two sets should be determined on the scores made by individuals who have attained some degree of mastery over both sets rather than over but one. Finally, the system of weights should be derived from the eighth-grade scores because those scores represent the final achievement, under the present school organization, resulting from formal training in arithmetic.

In Table IV the system of weights is shown and the method of deriving them is indicated. In the first horizontal column of the

TABLE IV

DERIVATION OF SYSTEM OF WEIGHTS

Set	A	B	C'	D	E	F	G	H	I	J	K	L	M	N	O
Eighth-grade score	28.9	25.8	19.9	22.8	8.0	10.6	6.7	8.6	4.7	6.1	11.4	4.4	5.4	2.4	5.2
Time allowance in seconds	30	30	30	30	30	60	60	30	60	120	120	180	180	180	180
Score per 30 seconds	28.9	25.8	19.9	22.8	8.0	5.3	3.35	8.6	2.35	1.53	2.85	.73	.90	.40	.87
Weight (relative difficulty)	1.0	1.12	1.45	1.27	3.61	5.45	8.63	3.36	12.3	18.9	10.1	39.5	32.1	72.2	33.3

table are the average eighth-grade scores for the 15 sets; in the second are the time allowances in seconds; in the third is the average score per 30 seconds for each set; and in the fourth are the

weights, or measures of relative difficulty. However, since the time allowance varies from set to set, the system of weights as presented in this table requires some revision. For, as the weights now stand, each of the sets with time allowances of 3 minutes has just six times as much influence in determining the total score as has any one of the sets with time allowance of 30 seconds. It is therefore necessary, in order that each set may have the same influence on the total score as any other set, to modify the weights to that end. This revised system of weights appears in Table V. The

TABLE V

EQUATION OF TIME ALLOWANCES

Set	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Weight (relative difficulty)	1.0	1.12	1.45	1.27	3.61	5.45	8.63	3.36	12.3	18.9	10.1	39.5	32.1	72.2	33.3
Time allowance in seconds	30	30	30	30	30	60	60	30	60	120	120	180	180	180	180
Weight after equating time allowances.....	1.0	1.12	1.45	1.27	3.61	2.73	4.31	3.36	6.15	4.73	2.53	6.58	5.35	12.0	5.56

same result would have been secured if in Table IV the time allowance had been neglected entirely and the weights computed on the average scores as they stood. Such a method, however, would have been misleading. As shown by the two steps taken in evolving the systems of weights, it now does two things: (1) it equates the examples on the basis of difficulty, and (2) it equates the time allowances of the several sets.

THE USE OF THE TEST

In a previous chapter it was pointed out that the test was evolved for the purpose of diagnosing weaknesses of one sort and another in school systems, schools, classes, and individuals. The method by which the test may be used for doing this thing will here be demonstrated.

We shall first make some comparisons between two large city systems—Cleveland and Grand Rapids. By using the system of weights just described, it is possible to get a single score to represent the arithmetical attainments of each grade for each of these two cities. These scores are given in Table VI and graphically represented in Diagram 2.

Turning to the diagram, we see that there are considerable differences between the two curves, especially in the lower grades.

TABLE VI

COMPARISON OF TOTAL SCORES MADE BY GRADES 3–8 IN CLEVELAND AND GRAND RAPIDS

City	Grade					
	3	4	5	6	7	8
Cleveland.........	90	181	260	318	370	430
Grand Rapids......	22	132	243	325	383	439

The superior attainment of the Cleveland children in the lower grades indicates relatively greater stress on arithmetic in this period. Work in arithmetic is begun earlier in Cleveland than in Grand Rapids, but this initial advantage is not maintained. From the showing that the latter city has made, the conclusion would seem to be justified that a large expenditure of time on arithmetic in the lower grades is of comparatively little importance in securing high attainment in the eighth grade.

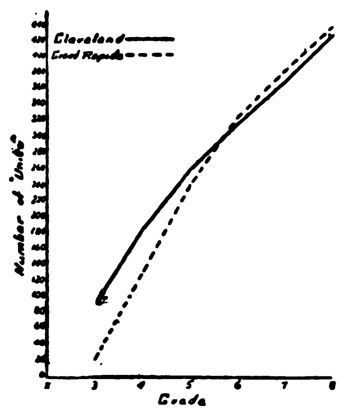


DIAGRAM 2.—Comparison of total scores made by Grades 3–8 in Cleveland and Grand Rapids.

However, this is a very general result and by itself is of little value because its meaning is vague and uncertain. The important thing for either city to know is its weak points. The detailed records by which this is possible are found in Tables VII and VIII and Diagrams 3, 4, 5. In Diagram 3 there appears a graphic comparison of the records made by the pupils of the two systems in each of the four sets in addition, A, E, J, M. The first glance at these curves shows that the relations between the two cities are about the same here as indicated by the general results, viz., superiority of Cleveland in the lower, and superiority of Grand

Rapids in the upper, grades. In the simple addition combinations, Set A, there is little difference between the two systems, except in

TABLE VII

Median Scores in Each Arithmetic Test for "B" Sections of Grades 3–8, Cleveland

Test	Grade					
	3	4	5	6	7	8
A	13.4	17.8	22.2	24.8	26.7	27.5
B	9.3	13.4	17.2	19.8	21.5	26.0
C	6.5	12.0	15.5	16.6	17.7	19.0
D	6.3	12.4	15.7	18.5	20.8	22.5
E	4.3	5.3	6.3	6.8	7.5	7.8
F	2.0	4.9	6.7	7.5	8.6	10.1
G	2.0	3.9	5.2	5.5	5.9	6.6
H	...	...	5.0	5.5	7.7	8.5
I	0.6	1.1	2.0	3.1	4.0	4.7
J	1.9	3.2	4.0	4.4	4.9	5.7
K	...	4.0	6.8	8.5	10.1	12.5
L	...	1.7	2.5	2.8	3.2	3.9
M	1.4	2.5	3.2	3.8	4.4	5.1
N	...	0.8	1.3	1.7	2.0	2.6
O	...	...	...	3.1	4.1	5.5

TABLE VIII

Median Scores in Each Arithmetic Test for Grades 3-1–8-2. Grand Rapids

Test	Grade											
	3-1	3-2	4-1	4-2	5-1	5-2	6-1	6-2	7-1	7-2	8-1	8-2
A	11.8	13.4	13.6	16.4	20.3	21.5	22.8	25.0	26.5	27.3	29.5	30.3
B	6.3	8.4	9.1	12.1	14.7	15.9	16.8	19.1	21.3	20.7	22.8	25.5
C	...	...	7.1	11.3	13.7	14.0	15.5	17.0	17.7	18.8	49.3	20.7
D	...	...	6.9	10.4	12.5	14.3	15.5	16.9	18.4	19.7	20.5	23.0
E	...	...	4.1	4.6	5.2	5.4	6.0	6.6	7.2	7.2	7.8	8.1
F	...	...	2.8	4.1	6.0	6.5	7.1	8.0	9.3	9.6	10.3	11.0
G	...	...	2.2	3.3	4.0	4.9	5.3	5.6	6.1	6.1	6.7	6.8
H	...	...	...	...	...	6.3	6.2	6.5	9.0	7.8	8.6	8.8
I	...	...	0.7	0.9	1.3	1.4	2.3	3.0	3.8	4.1	4.0	4.7
J	...	...	...	2.8	3.4	3.7	4.1	4.5	5.4	5.3	5.7	6.5
K	...	...	...	...	3.0	4.3	5.4	6.5	7.5	8.8	9.7	10.3
L	...	...	...	...	2.3	2.9	3.3	3.6	4.3	4.5	4.9	4.9
M	...	...	...	2.3	3.0	3.6	4.3	4.5	4.9	5.0	5.7	5.7
N	...	...	...	...	0.7	0.8	1.1	1.4	1.7	1.8	2.0	2.3
O	...	...	...	...	...	...	3.5	3.6	3.9	4.6	5.5	4.8

the eighth grade, where Grand Rapids clearly takes the lead. In the short-column addition Grand Rapids is comparatively weak, while in the more complex types of addition, Sets J and M, involving the

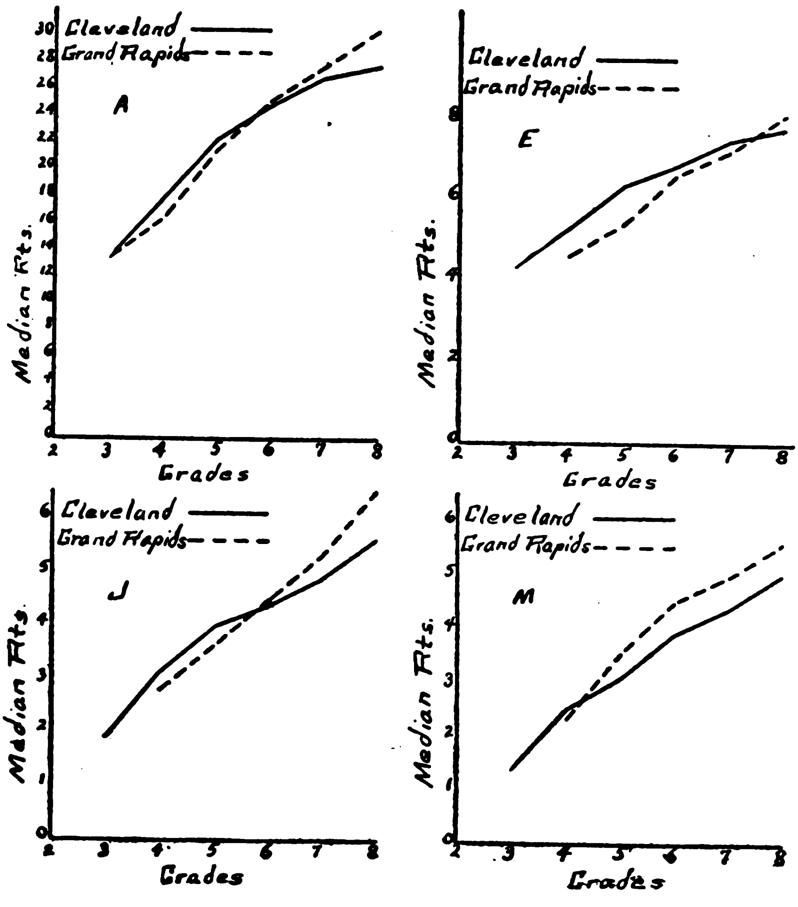


DIAGRAM 3.—A comparison of records made in Sets A, E, J, and M (addition) by Grades 3–8 in Cleveland and Grand Rapids.

bridging of the attention spans and "carrying," the superiority of that city is beyond question.

In Diagram 4 we have the same comparisons for the three sets in multiplication, C, G, and L. The facts here tend to confirm the previous statements concerning the relative standings of the two

cities. It should be added, however, that the characteristic difference is somewhat accentuated in Set L, the multiplication of four-place numbers by two-place numbers, a type of operation in which Cleveland appears to be particularly weak.

Passing to Diagram 5, conditions are found to be completely reversed. The test has revealed the weakness of the children of Grand Rapids in the fundamentals—division. In every one of the sets in division, D, I, K, and N, the Cleveland scores appear to advantage, but it is in the two sets in long division, K and N, that this relative excellence is most marked. The Grand Rapids children have not mastered the technique of long division.

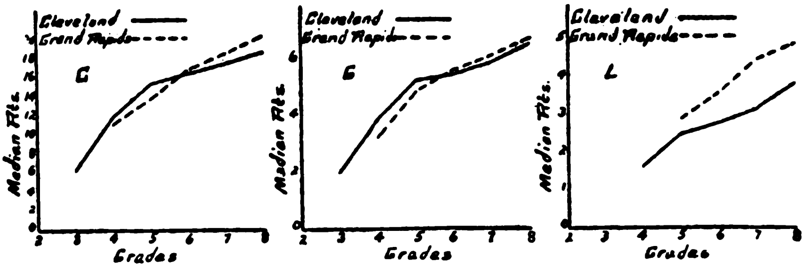


DIAGRAM 4.—Comparison of records made in Sets C, G, and L (multiplication) by Grades 3–8 in Cleveland and Grand Rapids.

In subtraction and fractions the characteristic difference is again apparent; comment on the records made in the sets involving these operations is therefore unnecessary. Enough has been said to indicate how the weaknesses in a school system may be detected.

As an instrument in the hands of the supervisor the test is of decided value. For the purpose of diagnosing class weaknesses the records made in the several sets may be graphically represented as in Diagram 6, a form of graph used by Mr. Courtis. In this diagram the horizontal lines represent the 15 sets of the test, the vertical lines the grades, and the irregular lines the records made by the three classes in Grade 6-2 in three Grand Rapids schools, Turner, Lafayette, and East Leonard. The figures at the points of the intersection of the lines are the average scores for Cleveland and Grand Rapids made in the indicated sets by the indicated grades. Thus, if a particular sixth-grade class should make scores

that would exactly agree in every case with this general average, the graphical representation of the record made by that class would coincide with the heavy black vertical line representing the sixth grade in the diagram. Deviations from this line indicate deviations

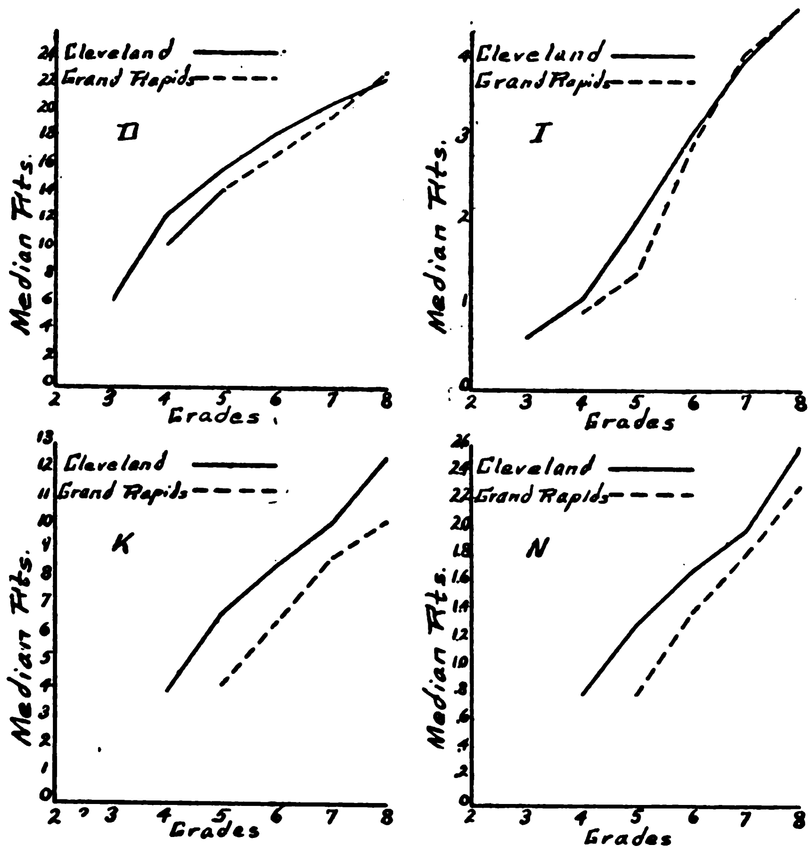


DIAGRAM 5.—A comparison of records made in Sets D, I, K, and N (division) by Grades 3–8 in Cleveland and Grand Rapids.

either above or below the Cleveland–Grand Rapids average, depending on whether such deviations are to the right or to the left.

Now on examining the diagram there is little tendency observed on the part of these three sixth-grade classes represented to follow the general average as indicated by the heavy vertical line. The

East Leonard School, represented by the broken line, is seen to be doing work of a relatively high order, since it drops below the average in no set. In Sets C (simple multiplication combinations), H (fractions), and O (fractions) this class is especially strong, while

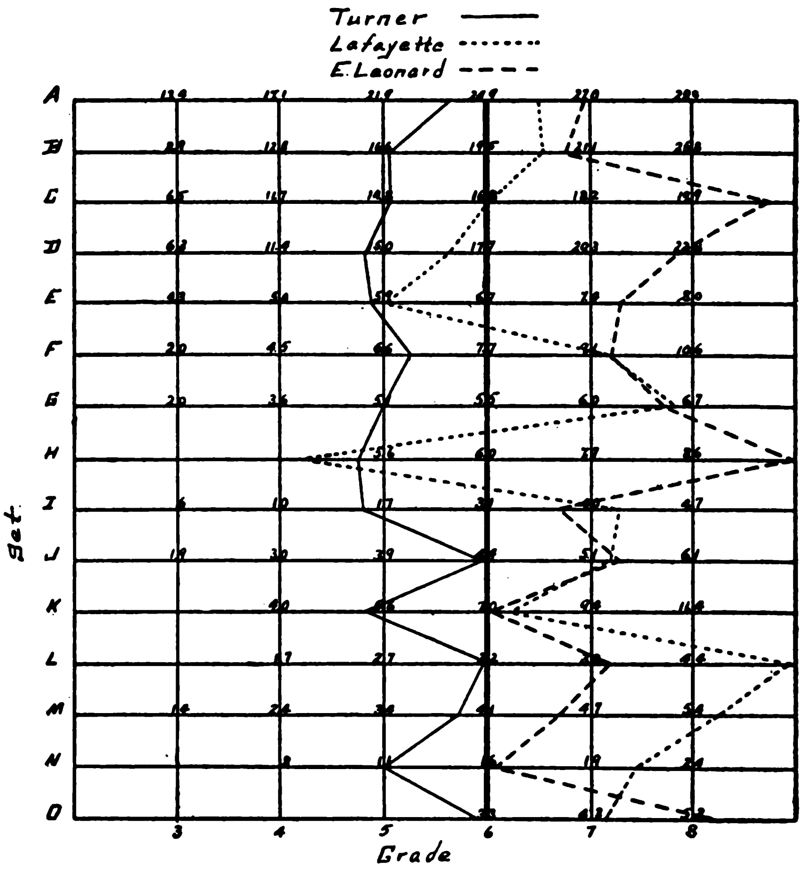


Diagram 6.—A comparison of the records made in each of the fifteen sets by the sixth grades in three Grand Rapids schools—Turner, Lafayette, and East Leonard.

its greatest weaknesses seem to be in division, I, K, and N, the typical Grand Rapids weakness. The Turner School, represented by the solid line, is seen to be of an entirely different type. Its work is consistently of a low order, being below the average in every set. The other class gives evidence of very poor supervision. Note the

erratic character of the curve. The class is very weak in short-column addition, E, and exceptionally strong in the addition of 5 four-place numbers; very poor in one type of fractions, H, and very good in another, O; but good in both of the more complex

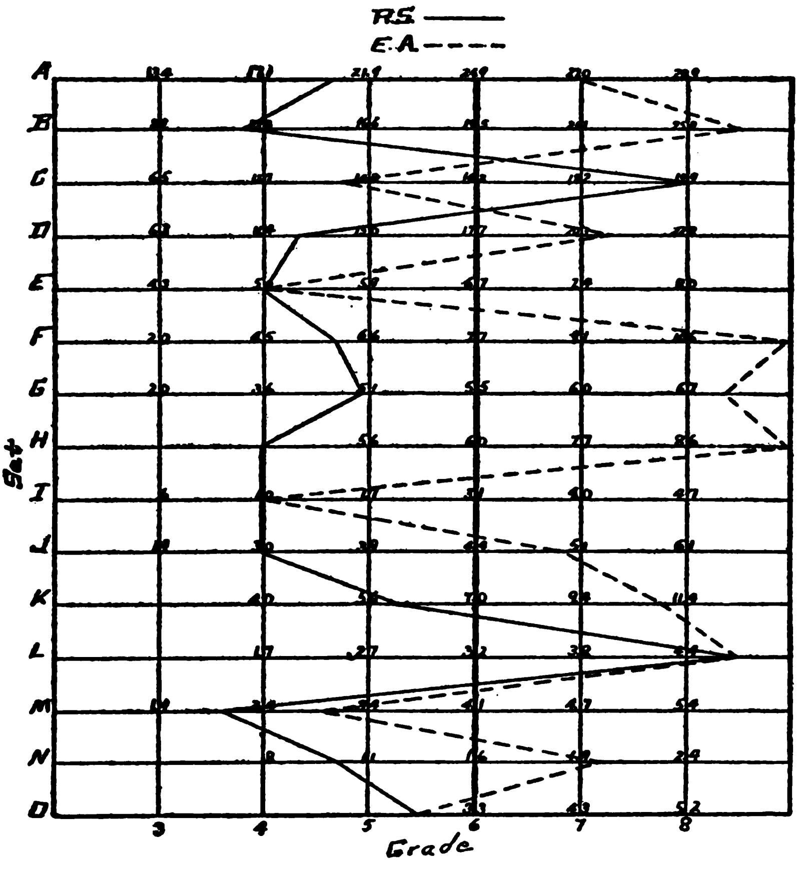


DIAGRAM 7.—A comparison of the records made in each of the fifteen sets by two sixth-grade pupils in the same class in Grand Rapids.

types of multiplication, G and L. Many more facts could be pointed out, but these will suffice to show how the test may be used to enable the supervisor to discover class weaknesses.

We come now to the use of the test by the teachers in studying the peculiarities of the individual pupil. In Diagram 7, identical

in form with the immediately preceding diagram, is presented the records of two sixth-grade pupils in the same class. The pupil R. S. is seen to be weak in everything except Sets C and L, two sets in multiplication. The pupil E. A., on the other hand, is strong in everything except the multiplication tables, C, short-column addition, E, short division, I, the complex addition, M, and fractions, O. With this record before her the teacher would be able to direct the pupils' time and energy into the needed channels and expend her own to a rational end.

DISTRIBUTIONS

A measure of central tendency, when used to represent a group, is always subject to the criticism that it is a *single* measure and gives no idea of the variations from this central tendency of the individuals composing the group. It is therefore necessary, in order that we may get a complete picture of the scores made by all the individuals composing the group, to present the entire distribution of the individual records.

If the distribution is to be valid, it is absolutely essential that there be no mistakes made in the timing of the individual pupils forming the distribution. When determining a measure of central tendency, as pointed out earlier in the chapter, it is merely necessary that no cumulative error be made, since the chance errors, occurring as often on the one side of the central tendency as on the other, offset one another. In the case of the distribution, on the other hand, it is obvious that these chance errors flatten out the distribution, since through error individual scores would be shifted to the one side or the other of this central tendency and thus decrease the actual frequency at that point. For this study of distribution, therefore, only the records made by pupils examined by the writer will be used. The number of pupils in each grade thus tested in the schools of Grand Rapids appears in Table IX. The number of cases is not great for any grade, but the records are accurate.

In Tables X, XI, XII, and XIII are presented the distributions for each grade in each of the four sets in addition, A, E, J, and M.

It will be noted that the distributions have been reduced to a percentage basis in order that the results from grade to grade may be strictly comparable.

TABLE IX

NUMBER OF PUPILS IN EACH GRADE TESTED BY THE WRITER IN THE GRAND RAPIDS SCHOOLS

Grade	3-1	3-2	4-1	4-2	5-1	5-2	6-1	6-2	7-1	7-2	8-1	8-2	Total
Pupils	31	30	38	123	70	108	166	193	135	174	176	132	1,376

Since the interpretation of the facts as they appear in the tables is comparatively difficult, the same facts for the upper sections of the grades are presented graphically in Diagram 8. The diagram is so constructed that movement down the graph from top to bottom is in line with progress through the grades, while movement from section to section across the diagram means movement from a simple type of operation to a more complex type. If this explanation be kept in mind some very interesting and significant relations may be noted in addition to the general fact that the curves on the whole indicate a normal distribution, that is, the largest number of cases being near the central point of the distribution with a symmetrical decrease on either side to zero.

A fact very clearly brought out by the diagram is that in each of the four sets the distribution curve tends to become flattened with progress through the grades. With few exceptions the curves for both the "attempts" and the "rights" become flattened with each successive grade. This means that the training received in the schools tends to accentuate individual differences rather than the contrary. While there is general progress of a particular type through the grades, the individuals of the grade in many instances depart from this typical rate of progress. The methods used work well with some individuals, but scarcely at all with others; and it is probably true that with the same training the bright pupil, while making relatively the same progress, makes absolutely much more than the dull pupil. Thus the individual variation is increased, while progress is made in both cases.

Another fact brought out in the diagram is that the curves also become flattened as we proceed from left to right across the graph, that is, from the less complex to the more complex types of addition.

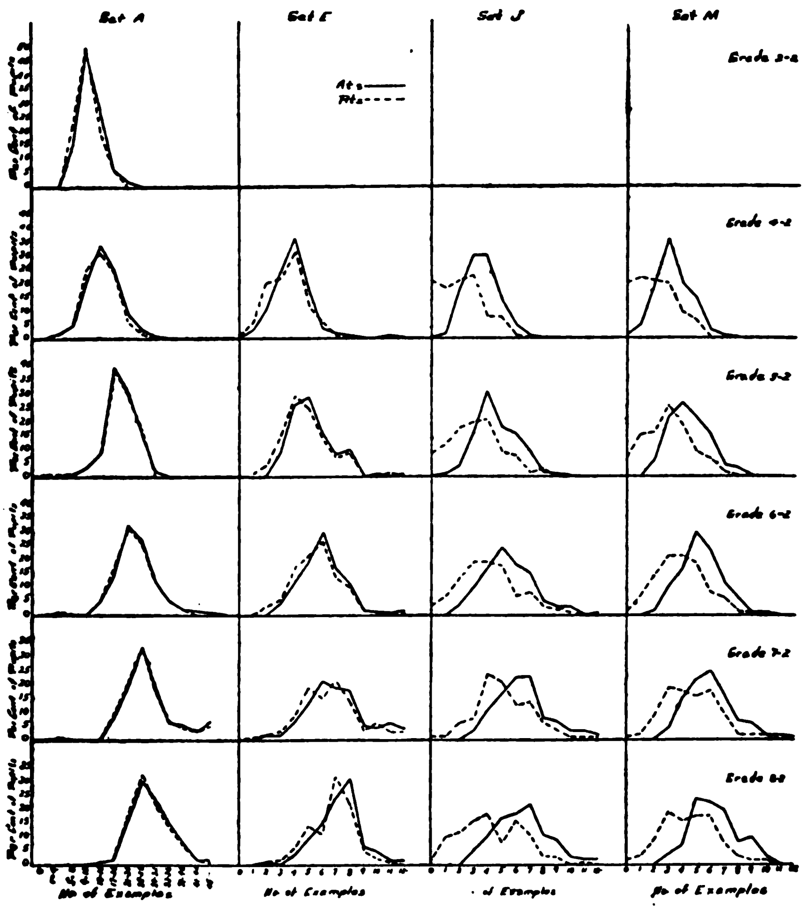


DIAGRAM 8.—A comparison of the distribution of "attempts" and "rights" in four sets in addition (A, E, J, M) for Grades 3–8.

This is not especially true in going from Set J to Set M, but it must be remembered that these are two *different* types of addition, the one involving the bridging of the attention spans and the other "carrying." Which of these two types is the more complex it

TABLE X

DISTRIBUTION OF 100 PUPILS IN EACH GRADE—SET A

		Score															
		0	1–4	5–8	9–12	13–16	17–20	21–24	25–28	29–32	33–36	37–40	41–44	45–48	49–52	53–56	57–60	61–65
3-1	Attempts	....	....	3	44	40	7	3	3	....	....	....	....	....	....	....	....	....
	Rights	....	....	3	58	33	3	3	....	....	....	....	....	....	....	....	....	....
3-2	Attempts	....	....	15	48	28	7	2	....	....	....	....	....	....	....	....	....	....
	Rights	....	....	23	50	20	7	....	....	....	....	....	....	....	....	....	....	....
4-1	Attempts	....	....	8	34	42	11	5	....	....	....	....	....	....	....	....	....	....
	Rights	....	....	8	37	40	10	5	....	....	....	....	....	....	....	....	....	....
4-2	Attempts	....	2	5	21	34	25	9	3	1	....	....	....	....	....	....	....	....
	Rights	....	2	5	25	32	26	7	2	1	....	....	....	....	....	....	....	....
5-1	Attempts	....	....	1	7	31	33	25	2	1	....	....	....	....	....	....	....	....
	Rights	....	....	1	10	35	33	20	....	1	....	....	....	....	....	....	....	....
5-2	Attempts	....	....	1	3	8	39	31	15	2	....	1	....	....	....	....	....	....
	Rights	1	1	1	3	8	38	30	17	....	....	1	....	....	....	....	....	....
6-1	Attempts	....	....	....	1	6	21	31	20	8	4	4	2	2	1	....	....	....
	Rights	....	....	1	1	7	22	30	19	7	4	6	1	1	1	....	....	....
6-2	Attempts	....	....	....	....	5	14	32	27	12	5	2	....	1	....	1	....	1
	Rights	....	1	....	....	5	17	31	26	12	5	2	....	....	....	1	....	....
7-1	Attempts	....	....	....	1	1	13	27	30	13	7	4	1	2	1	....	....	....
	Rights	....	....	....	2	3	13	24	30	15	6	4	2	1	....	....	....	....
7-2	Attempts	....	....	....	....	....	8	19	33	19	6	5	3	1	4	1	....	1
	Rights	....	1	....	....	....	9	20	33	18	7	4	3	1	3	....	....	1
8-1	Attempts	....	....	....	....	....	1	16	31	24	11	11	2	2	1	1	....	....
	Rights	....	....	....	....	....	2	15	34	22	11	11	2	2	1	....	....	....
8-2	Attempts	....	....	....	....	1	2	16	30	24	15	8	2	....	....	....	1	1
	Rights	....	....	....	....	1	2	18	33	22	14	7	2	....	....	....	1	....

TABLE XI

DISTRIBUTION OF 100 PUPILS IN EACH GRADE—SET E

		Score																
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
4-1	Attempts			10	26	40	18	3	3									
	Rights		8	26	26	24	13	3										
4-2	Attempts		3	11	24	36	18	4	2	1				1				
	Rights	1	6	20	22	31	12	6	1				1					
5-1	Attempts			4	12	41	28	7	4	4								
	Rights	1	3	7	22	30	26	6	4	1								
5-2	Attempts				9	26	29	16	8	10		1		1				
	Rights		1	4	15	29	20	14	7	9			1					
6-1	Attempts			2	4	14	16	29	15	12	5	2	1					
	Rights	1		3	6	17	15	29	14	8	4	2	1					
6-2	Attempts				4	12	19	30	17	12	2	1	1	1	1			
	Rights			3	5	17	21	25	13	1	2	1	1		1			
7-1	Attempts				1	16	15	29	16	17	3	3						
	Rights		1	2	6	16	16	30	14	10	3	2						
7-2	Attempts			2	2	6	13	21	19	18	5	4	6	2				2
	Rights		1	2	3	9	19	15	22	14	3	6	3	1		1		1
8-1	Attempts			1	1	1	11	17	24	26	6	6	2	3		1	1	
	Rights			1	2	4	14	17	27	18	6	5	2	3	1			
8-2	Attempts			1		4	9	15	24	31	7	5	2		1	1		
	Rights			1	2	6	14	11	32	22	5	3	2	1		1		

would be difficult to say. However, in the other cases the statement is certainly true. This would indicate less individual

TABLE XII

DISTRIBUTION OF 100 PUPILS IN EACH GRADE—SET J

	Score														
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
4-2 Attempts.	....	2	18	30	30	14	5	1	....	....	....	....	....	....	....
4-2 Rights....	21	18	21	23	8	8	....	1	....	....	....	....	....	....	....
5-1 Attempts.	....	....	10	20	41	17	7	2	3	....	....	....	....	....	....
5-1 Rights....	9	17	17	27	14	13	2	1	....	....	....	....	....	....	....
5-2 Attempts.	....	1	4	17	31	18	16	9	2	1	....	....	....	1	....
5-2 Rights....	8	12	18	20	21	9	8	1	3	....	....	....	....	....	....
6-1 Attempts.	....	....	2	10	22	28	16	11	7	3	....	1	....	....	....
6-1 Rights....	2	7	10	28	21	11	8	6	5	2	....	....	....	....	....
6-2 Attempts.	....	....	4	10	17	24	18	15	5	3	3	....	....	1	....
6-2 Rights....	3	7	13	19	19	18	7	8	3	2	1	....	....	....	....
7-1 Attempts.	....	....	1	9	19	24	16	14	5	5	2	2	2	....	1
7-1 Rights....	4	8	12	13	22	15	8	9	3	2	2	1	1	....	....
7-2 Attempts.	....	....	....	3	11	17	23	23	8	7	3	3	....	1	1
7-2 Rights....	1	1	6	8	24	21	13	14	6	3	1	1	1	....	....
8-1 Attempts.	....	....	1	2	10	19	19	18	14	8	5	1	2	....	1
8-1 Rights....	2	4	7	14	15	19	10	15	8	4	1	1	....	....	....
8-2 Attempts.	....	1	....	5	10	17	18	22	11	9	3	2	2	....	....
8-2 Rights....	2	10	12	16	18	8	16	11	3	3	....	1	....	....	....

TABLE XIII

DISTRIBUTION OF 100 PUPILS IN EACH GRADE—SET M

	Score												
	0	1	2	3	4	5	6	7	8	9	10	11	12
4-2 Attempts....	1	5	19	36	20	15	3	1	....	....	....	....	....
4-2 Rights.......	20	22	21	20	10	7	....	....	....	....	....	....	....
5-1 Attempts....	....	4	17	16	33	25	2	2	1	....	....	....	....
5-1 Rights.......	16	14	33	14	14	7	2	....	....	....	....	....	....
5-2 Attempts....	....	....	6	22	27	22	16	4	3	....	....	....	....
5-2 Rights.......	6	15	16	26	19	9	7	2	....	....	....	....	....
6-1 Attempts....	....	....	4	10	22	30	21	6	4	2	....	1	....
6-1 Rights.......	3	8	11	24	21	16	11	2	2	1	....	1	....
6-2 Attempts....	....	....	2	10	16	30	23	11	6	1	1	....	....
6-2 Rights.......	1	7	14	21	21	19	10	6		1	....	....	....
7-1 Attempts....	....	....	....	8	15	25	18	16	11	3	2	2	....
7-1 Rights.......	5	5	12	11	19	15	15	11	4	2	1	....	....
7-2 Attempts....	....	....	....	4	15	22	25	16	7	6	2	2	1
7-2 Rights.......	1	2	9	19	18	16	18	10	2	2	2	....	1
8-1 Attempts....	....	....	1	3	9	18	29	19	12	4	3	1	1
8-1 Rights.......	1	3	9	12	15	20	21	7	8	3	1	....	....
8-2 Attempts....	....	1	....	4	7	24	23	20	8	10	3	....	....
8-2 Rights.......	2	3	9	19	16	18	18	8	3	2	2	....	....

variation on the simple addition examples than on the more complex.

The character of the relation between the curves for the "rights" and for the "attempts" is a third matter deserving attention. On the average the curve for the "rights" is flatter than that for the "attempts." This is emphatically true in Sets J and M, the more complex types. Thus there is less tendency among the pupils to vary in the number of examples attempted than in the number solved correctly. This is probably explained by the fact that the number of examples attempted is controlled quite largely by the physical limitations on speed, since the character of the operation in each of these types of examples is familiar to all the pupils. In working the examples correctly, on the other hand, another factor is involved, and that is the factor of right and wrong associations. A more strictly mental limitation is here added to the physical limitation just mentioned.

TABLE XIV

DISTRIBUTION OF 100 PUPILS IN EACH GRADE—SET L

		Score								
		0	1	2	3	4	5	6	7	8
5-1	Attempts.....	......	*8*	*27*	*37*	*26*	*2*	......	......	......
	Rights.......	26	20	26	21	6	1	......	......	......
5-2	Attempts.....	......	......	*14*	*32*	*38*	*11*	*2*	*3*	......
	Rights.......	12	16	28	25	12	4	2	1	......
6-1	Attempts.....	......	*2*	*8*	*22*	*31*	*20*	*11*	*5*	*1*
	Rights.......	4	13	19	26	19	12	4	3	......
6-2	Attempts.....	......	*1*	*3*	*14*	*34*	*29*	*12*	*5*	*2*
	Rights.......	3	12	17	24	22	14	7	1	......
7-1	Attempts.....	......	*1*	*2*	*11*	*29*	*21*	*19*	*6*	*11*
	Rights.......	4	10	17	20	25	10	11	3	......
7-2	Attempts.....	......	......	*2*	*9*	*17*	*22*	*23*	*17*	*10*
	Rights.......	1	6	10	18	22	19	15	8	1
8-1	Attempts.....	......	*1*	*2*	*3*	*13*	*21*	*27*	*22*	*11*
	Rights.......	1	5	7	17	24	21	16	7	2
8-2	Attempts.....	......	......	*1*	*6*	*15*	*23*	*24*	*15*	*16*
	Rights.......	1	4	11	20	23	14	17	6	4

Since the foregoing characteristics are not peculiar to the distributions in addition, but are common to the distributions in each of the other three fundamentals as well, it is not necessary to present tables and graphs setting forth the distributions in these three

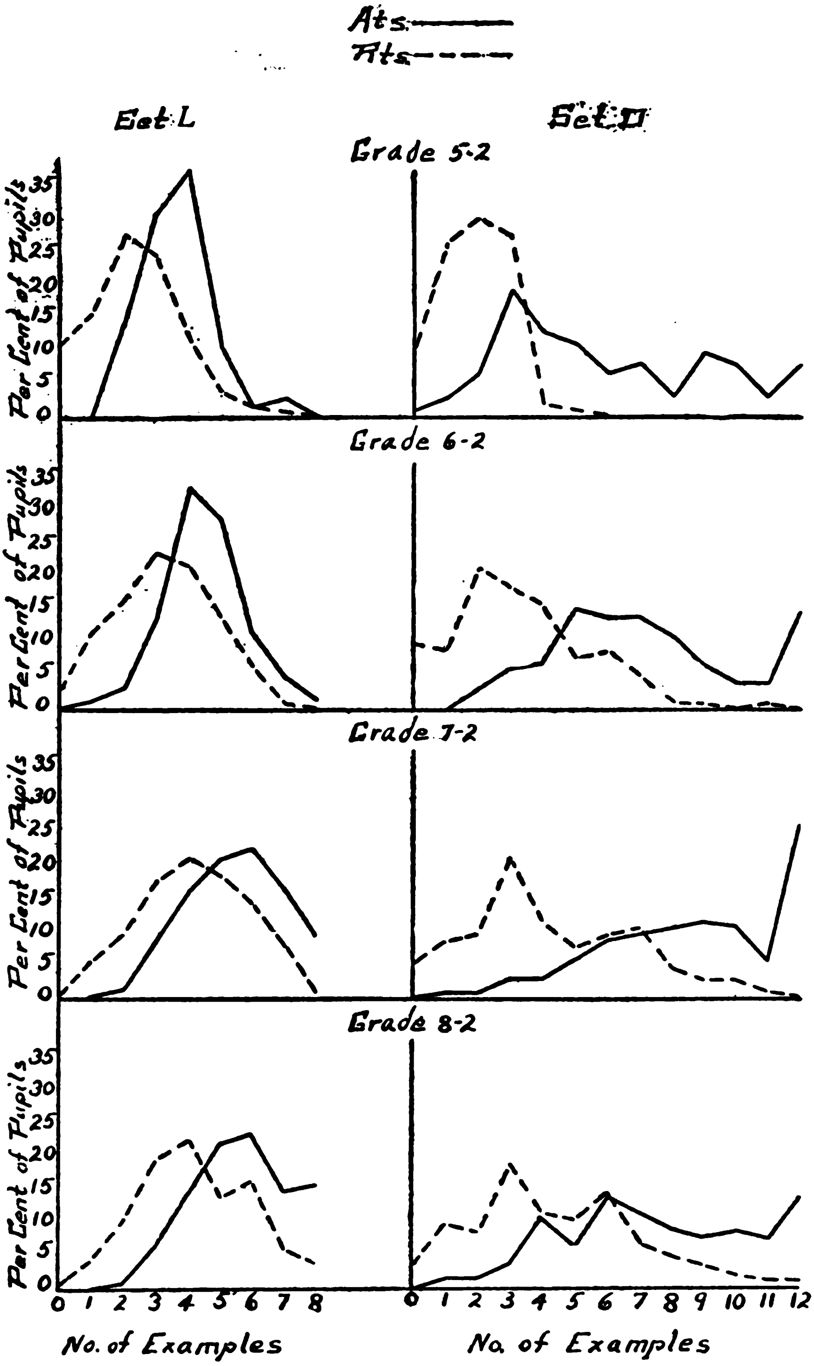
operations. We have an entirely different proposition in the case of fractions. For this reason, therefore, Tables XIV and XV are accompanied by Diagram 9, which graphically portrays the facts found in the tables. In the first table appear the distributions for the several grades in Set L, multiplication; in the second the distributions in Set O, fractions.

TABLE XV

DISTRIBUTION OF 100 PUPILS IN EACH GRADE—SET O

		Score												
		0	1	2	3	4	5	6	7	8	9	10	11	12
5-2	Attempts....	*1*	*3*	*7*	*20*	*13*	*11*	*7*	*8*	*3*	*10*	*7*	*3*	*7*
	Rights.......	11	27	31	28	2	1	....	....	....	....	....	....	....
6-1	Attempts....	....	*2*	*3*	*8*	*10*	*16*	*10*	*13*	*11*	*7*	*6*	*3*	*11*
	Rights.......	13	13	16	18	7	13	6	8	4	1	....	1	....
6-2	Attempts....	....	....	*3*	*6*	*7*	*15*	*14*	*14*	*11*	*7*	*4*	*4*	*15*
	Rights.......	10	9	22	18	16	8	9	5	1	1	....	1	....
7-1	Attempts....	....	*1*	*2*	*7*	*7*	*7*	*8*	*8*	*9*	*11*	*14*	*4*	*22*
	Rights.......	15	9	13	23	13	5	5	1	9	2	4	1	....
7-2	Attempts....	....	*1*	*1*	*3*	*3*	*6*	*9*	*10*	*11*	*12*	*11*	*6*	*27*
	Rights.......	6	9	10	22	12	8	10	11	5	3	3	1	....
8-1	Attempts....	....	....	*1*	*3*	*6*	*7*	*13*	*12*	*9*	*10*	*12*	*9*	*18*
	Rights.......	2	7	10	19	16	10	11	6	8	4	3	3	1
8-2	Attempts....	....	*2*	*2*	*4*	*11*	*7*	*14*	*12*	*9*	*8*	*9*	*8*	*14*
	Rights.......	4	10	9	19	12	11	15	7	5	4	2	1	1

The diagram is of the same order as the previous one and therefore requires no explanation. The similarity between the curves for Set N and those for the sets in addition just discussed is apparent. Let us therefore turn at once to a comparison of the curves of this set and of Set O. Perhaps the most obvious feature in the comparison is the relation between the curves for the "rights" and the curves for the "attempts" in Set O. Here, in direct contrast to the sets in the fundamentals, the curves for the "attempts" present a much more flattened appearance than the curves for the "rights." This would seem to indicate that, whereas in the fundamentals the knowledge of the character of the operation to be performed was common property for practically all the pupils, in fractions the character of the operation is not known by all. To those familiar with the method of handling fractions, or to those who think themselves familiar with it, it is a simple matter to attempt a



] GRAM 9.—A comparison of the distribution of "attempts" and "rights" in Set L (n ti on) and Set O (fractions) for Grades 5–8.

large number of the examples. This is not true of the fundamentals. For instance, take an example in long division. Even though the method of working such an example is perfectly familiar, it requires considerable time to work it because it is a long process. Speed can be developed only through much practice by making a large number of reactions quite automatic. The actual process involved in working an example in fractions, such as is found in Set O, is, on the other hand, a relatively short one. Now, since so far as attempting the examples is concerned it is just about as easy to attempt one of the examples as another, those pupils who are familiar with the method of solving fractions attempt a large number, or all of them. Those, on the other hand, who are unfamiliar with the method are able to attempt but a few. In this way the curve for "attempts" becomes flattened. The curve for "rights" is less flattened because of the composition of the test set. As will be pointed out later, the examples in the multiplication and division of fractions are easier than the other two types. This causes the distribution of "rights" to be largely confined to six examples. Since there is no such factor operating to narrow down the distribution of "attempts," the curve for the "rights" is elevated in comparison.

Another fact indicated by the diagram which bears somewhat on this same matter is the increase of the percentage of pupils attempting all the examples in Set O up to Grade 7-2 and then a decrease in the percentage to Grade 8-2. An examination of Table XV gives further evidence on this same point. It is seen that there is a constant increase in this percentage from Grade 5-2 through Grades 6-1, 6-2, and 7-1 to Grade 7-2, where the maximum of 27 per cent is reached. Then there is a decrease to 18 per cent in Grade 8-1, and a further decrease to 14 per cent in Grade 8-2. This is a very significant fact. An inspection of the work actually done by the pupils on this set indicates a tendency among them to substitute various "easy" methods for the correct methods in working the examples. For example, a pupil may add two fractions by adding their numerators and their denominators. This takes less time than the right method. Thus, by substituting invalid for valid methods the pupil is enabled to complete the set in a relatively

short time. As the pupil matures he gradually develops greater speed, and this probably accounts for the increase in the number of pupils attempting all the examples of the set up to Grade 7-2. The decrease from this point on is probably due to the weeding out of these invalid and short methods through increasing familiarity with fractions.

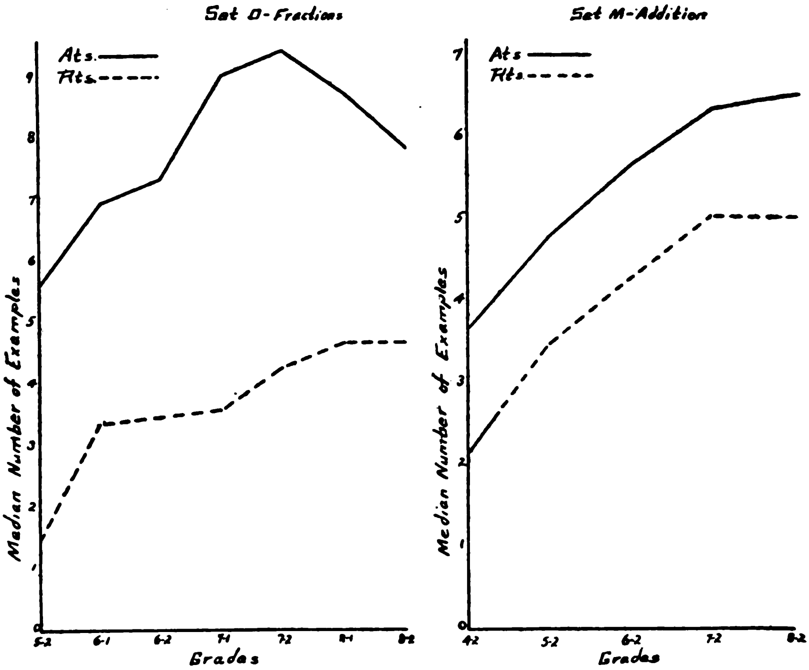


DIAGRAM 10.—A comparison of the median numbers of "attempts "and "rights" for the several grades in Set O (fractions) and Set M (addition).

At this point in the discussion Diagram 10 may be introduced, although it is not a diagram of distributions. But, since it bears out what has just been said, it will do no harm to examine it. There are two sections in the diagram, the one showing curves of median "attempts" and median "rights" through the grades for Set O, fractions; the other doing the same thing for Set M, addition. The striking fact brought out by the diagram is that the median number of examples attempted decreases from Grade 7-2, while the median

of "rights" either increases or remains stationary. As pointed out in the previous paragraph, this indicates the substitution of an easy invalid method for the valid one. All of this goes to show that the teaching of fractions is not a simple matter and should not be confused with the teaching of the fundamentals.

ACCURACY

Although the importance of standards of accuracy is clearly recognized, only a very brief study has been made of accuracy because of the immense amount of labor involved in the undertaking. However, a study has been made of 2,400 cases, 400 from each grade taken at random from the records made by the Cleveland children. The records were all scored by the writer, lest an error enter into the results due to the scoring by the pupils. The results of this study are found in Table XVI. But, since accuracy by itself, separated from a statement of the number of examples attempted, has but little meaning, this table is accompanied by Table XVII. In the latter appear the average number of examples attempted and the average number worked correctly by this same group of 2,400 pupils in each of the sets.

Turning now to Table XVI, it is noted that of the 4 sets in the simple combinations (A, B, C, D) Set C seems to be markedly the most difficult. It is also seen that from the third grade to the eighth, not only is there no increase in accuracy in this set, but there is an actual decrease. This is due to certain peculiar types of errors that the children make in this set. A complete discussion of these errors will be found in the following chapter. Incidentally this inaccuracy throws some light on a related matter. Reference to the standard score in Table II shows that the score for Set C is smaller than the score for Set D in the upper grades. It has been contended by some that this difference is to be accounted for by the fact that in writing the product in Set C two figures are required in most cases, while in Set D the quotients are all single figures. Now it is undoubtedly the case that this is a factor, but that it is not the only one is clearly shown by this table on accuracy in conjunction with the accompanying table on "rights" and "attempts," Table XVII. The latter table shows that the difference in scores

made in the two sets is largely due to inaccuracy, since there is but little difference in the number of examples attempted.

A further examination of Table XVI shows the greatest inaccuracy to be found in fractions (both sets), short division, the addition

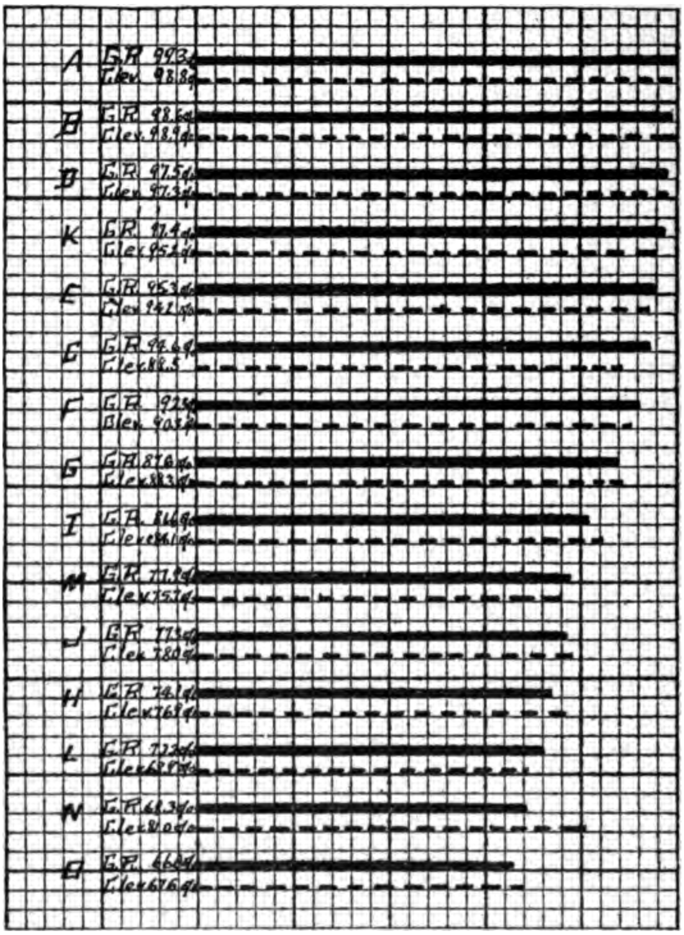


DIAGRAM 11.—A comparison of percentages of accuracy made in each of the fifteen sets by eighth-grade pupils in Cleveland and Grand Rapids.

of long columns, the addition involving "carrying," and the multiplication of four- by two-place numbers. The accuracy is quite high in long division, Sets K and N. This is probably due to the

frequent change in the type of mental operation demanded. Thus long mental strain is avoided.

In Diagram 11 the accuracy achieved in the various sets by Cleveland and Grand Rapids eighth-grade pupils is shown. Cleveland is represented by 400 cases and Grand Rapids by 150, taken at random. In general, the results from the two cities agree closely, the two more obvious exceptions being found in Sets C and N. In the former Grand Rapids is quite superior to Cleveland, while in the latter the reverse is true. It should be remembered in this connection that earlier in the chapter it was found that in median scores the weakness of Grand Rapids was found to be in long division. Thus it is seen that this weakness is further indicated by inaccuracy.

TABLE XVI

PERCENTAGE OF ACCURACY IN EACH SET FOR GRADES 3–8. DATA FROM 2,400 PUPILS

Set	Grade					
	3	4	5	6	7	8
A	95.6	98.5	98.9	98.8	98.8	98.8
B	91.9	96.3	98.0	98.1	98.2	98.9
C	89.8	90.8	90.5	88.4	87.4	88.6
D	83.8	93.4	95.3	97.0	97.2	97.3
E	87.2	91.0	92.1	93.5	93.0	94.3
F	56.9	76.1	87.5	88.2	87.3	90.4
G	61.4	81.3	85.4	86.9	85.7	88.4
H	46.9	74.1	68.4	68.6	73.5	76.9
I	28.0	46.6	68.1	75.8	80.3	84.2
J	54.8	67.3	73.8	76.8	75.8	78.0
K	31.8	78.8	88.1	90.3	92.0	95.2
L	....	48.5	59.1	62.8	62.5	68.9
M	45.5	63.7	66.9	69.9	73.0	75.7
N	....	29.1	53.9	58.6	65.1	81.0
O	....	16.4	12.1	52.6	58.0	67.6

The accuracy from grade to grade in 13 of the sets, as the facts were presented in Table XVI, is shown in Diagram 12. As would be expected, the diagram shows most progress to be made in the more complex types of examples. In the simple combinations there is very little progress. As a general rule there is consistent progress from grade to grade. To this statement the sets in fractions offer exceptions, as they do in a great many respects.

The reader has perhaps already noted the fact that the accuracy-curve is quite different in form from the curve of "rights." In

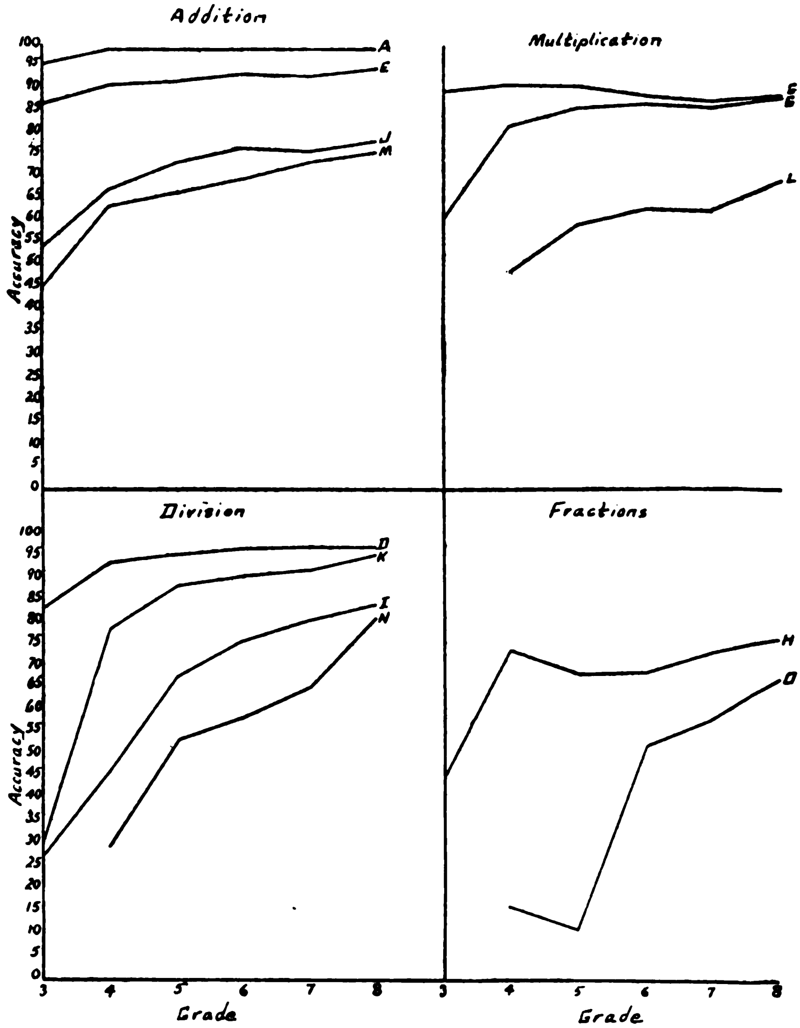


DIAGRAM 12.—Showing the progress in accuracy in each of thirteen sets through the grades.

Diagram 13 these two curves are compared for Set K. The curve for the "rights" possesses that quality so characteristic of the

Courtis standards—uniform progress from one grade to the next. The accuracy-curve, on the other hand, resembles the learning-curve, going up very rapidly at first and gradually turning over to

TABLE XVII

AVERAGE RIGHTS AND AVERAGE ATTEMPTS MADE IN EACH SET. GRADES 3–8, 2,400 PUPILS

Set	Grade											
	3		4		5		6		7		8	
	Rights	Attempts	Rights	Attempts	Rights	Attempts	Rights	Attempts	Rights	Attempts	Rights	Attempts
A........	16.3	*17.0*	20.3	*20.6*	23.3	*23.6*	25.2	*25.5*	28.1	*28.4*	28.8	*29.1*
B	9.9	*10.8*	14.0	*14.5*	18.3	*18.7*	20.1	*20.5*	22.4	*22.8*	25.8	*26.0*
C........	7.2	*8.1*	13.5	*14.9*	15.7	*17.4*	17.4	*19.6*	18.7	*21.4*	19.0	*21.5*
D.......	6.7	*8.0*	13.1	*14.0*	16.6	*17.4*	19.3	*19.9*	21.8	*22.4*	22.8	*23.5*
E	4.4	*5.1*	5.7	*6.2*	6.2	*6.7*	6.5	*7.0*	7.5	*8.0*	7.7	*8.2*
F........	2.1	*3.7*	4.6	*6.1*	6.9	*7.8*	7.5	*8.5*	8.5	*9.8*	9.6	*10.7*
G.......	1.6	*2.5*	3.5	*4.3*	4.7	*5.5*	5.2	*6.0*	5.8	*6.7*	6.2	*7.0*
H.......	0.8	*1.7*	2.6	*3.5*	4.2	*6.1*	6.4	*9.3*	8.1	*11.0*	8 8	*11.4*
I........	0.4	*1.5*	1.0	*2.1*	1.9	*2.8*	2.8	*3.6*	3.6	*4.5*	4.3	*5.1*
J........	1.7	*3.0*	3.1	*4.6*	3.7	*5.0*	4.3	*5.6*	4.8	*6.3*	5.5	*7.1*
K.......	0.02	*0.06*	2.7	*4.7*	6.1	*7.0*	8.2	*9.1*	10.1	*11.0*	12.0	*12.6*
L........	.....	*0.1*	1.4	*3.0*	2.1	*3.6*	2.4	*3.8*	2.8	*4.5*	3.4	*5.0*
M.......	0.9	*2.1*	2.4	*3.8*	2.9	*4.3*	3.5	*5.0*	4.2	*5.8*	4.7	*6.2*
N.......	.....	.....	0.4	*1.3*	0.9	*1.7*	1.2	*2.0*	1.6	*2.4*	2.2	*2.7*
O	.....	.....	0.1	*0.4*	0.2	*1.2*	3.9	*7.5*	4.7	*8.2*	6.0	*8.8*

the horizontal. From this comparison it would seem that accuracy is attained as the result of practice, thus approaching the learning-curve, while the development of speed is dependent on the maturing of the pupil.

SUMMARY

1. Standard scores for the several sets in Grades 3–8 have been determined on the basis of results secured from Cleveland and Grand Rapids pupils. A comparison of these scores with the Courtis standard scores in sets A, B, C, and D indicates that the scores in these sets constitute quite accurate standards of attainment; and there seems to be no reason for believing that the scores

in the other Sets, with the possible exception of set H, do not constitute equally accurate standards.

2. A system of weights has been derived whereby it is possible to equate the scores made in the several sets so that a single score may be secured to represent the general arithmetical attainment of an individual or group.

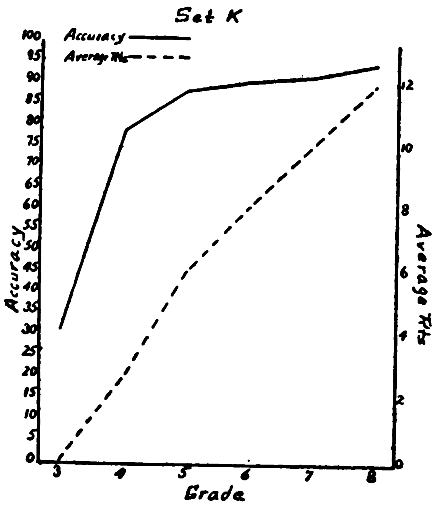


DIAGRAM 13.—A comparison of curves of accuracy and "rights" for Set K (long division).

3. The use of the test is considered at some length. Methods of diagnosing individual, class, school, and city weaknesses are indicated.

4. Some very interesting facts are brought out in comparing grade distributions in the various types of examples. First, in the fundamentals the distribution-curve tends to become flattened with progress through the grades. Secondly, the distribution-curve also tends to become flattened as we proceed from the less complex to the more complex types of examples in the fundamentals. Thirdly, as a general proposition in the fundamentals the distribution-curve representing the "rights" is flatter than that representing the "attempts." Fourthly, in Set O, fractions, the exact reverse of this last statement is true, the curve for the "attempts" being flatter than that for the "rights."

5. Tentative standards of accuracy for each of the sets in Grades 3–8 inclusive have been determined on the basis of results from Cleveland and Grand Rapids children.

6. Curves representing progress in accuracy through the grades and curves representing progress in the average number of examples worked are compared. The accuracy-curve takes the form of the learning-curve, while the "rights"-curve does not.

CHAPTER IV

TYPES OF ERRORS

This study represents an attempt to discover the different types of errors made in the various sets by pupils. In order to keep the study within limits it has been confined almost entirely to the eighth grade, a few comparisons being made between the eighth and fifth grades in the simple combinations. Furthermore, it has been found impossible to make a study of the errors made in certain of the sets, because of the impossibility of isolating them. For instance, it is impossible to determine beyond a reasonable doubt from the record made by a pupil in working an example in Set M, addition of 5 four-place numbers, whether or not an error in the sum is due to a mistake made in adding or "carrying." For like reasons no attempt is made to study errors made in Sets E, F, G, I, and J. It is therefore apparent that this study should be supplemented by experimentation.

ADDITION

For the reason just stated Set A is the only set in addition that can be profitably studied. Certain facts relating to errors made in this set appear in Tables XVIII and XIX and in Diagram 14. In the first table there is a comparison of the distributions of 100 errors made in the first 26 simple addition combinations of the set by Cleveland and Grand Rapids eighth-grade pupils. It will be noted that these 26 combinations include the first two rows of examples. In order to determine the distribution of 100 errors for these combinations, only those records of the eighth-grade pupils for each city were selected in which the pupils had attempted all the examples in the first two rows. That is, no record was used which showed that the pupil had not attempted every one of these 26 examples. This method makes the numbers of errors for the 26 combinations strictly comparable. Thus, turning to the table, it is understood if read as follows: Of the 100 errors made by the Cleveland eighth-grade pupils on the 26 combinations, all of which were attempted

the same number of times, none was made on the first combination, 1+2; six were made on the second, 6+6, and so on. In the second table the fifth and eighth grades of Grand Rapids are compared in the same way as were the two eighth grades in the first table, with the exception that the 100 errors are distributed over but 15 combinations. In Diagram 14 are reproduced certain interesting and typical errors actually made by the pupils.

An examination of the combined results from Cleveland and Grand Rapids in Table XVIII shows the easiest combinations to be 1+2, 7+7, 0+7, and 3+1. With the exception of the second of these combinations, the sum is in each case less than ten. For

TABLE XVIII

A Comparison of the Distributions of 100 Errors Made in 26 Simple Addition Combinations by Eighth-Grade Pupils in Cleveland and Grand Rapids

City	Simple Addition Combinations																										Total
	1 2	6 6	9 5	0 1	4 2	1 3	7 7	9 6	3 0	2 4	1 5	3 8	6 9	0 7	3 2	8 1	9 9	7 6	8 0	2 5	1 6	4 7	8 9	0 5	2 7	3 1	
Cleveland	...	6	9	2	2	1	...	6	3	4	4	5	4	1	8	2	3	7	1	1	5	6	7	1	11	1	100
Grand Rapids	...	5	4	2	2	1	...	9	...	...	17	7	11	...	12	1	5	4	1	1	4	3	2	1	8	...	100
Total	...	11	13	4	4	2	...	15	3	4	21	12	15	1	20	3	8	11	2	2	9	9	9	2	19	1	200

some reason the association, 7+7=14, is very strong. There is even a tendency, as will be pointed out in the discussion of Set C, to add 7 and 7 when the combination appears in a set of examples in multiplication. A further curious fact is that the three combinations 1+5, 3+2, and 2+7 are the most difficult, or rather net the most errors, and yet their sums are also less than ten in each case. And there seems to be a fixed association between each of these and a sum which is incorrect. The typical associations for these combinations when errors are made appear in sections *b*, *c*, and *d* of Diagram 14. There is a strong tendency to say 3+2=6, 1+5=7, and 2+7=8. These three errors account for practically all the errors made in these combinations. However, with these exceptions errors are made more frequently with the larger than with the smaller combinations. Of the 200 total errors considered, the ten combinations whose sums are greater than ten show the average

number of errors made per combination to be 10.3, while the average number for the sixteen combinations with sums less than ten is but 6.1. Now, if the combination 7+7 be eliminated from the first group, and the combinations 5+1, 3+2, and 7+2 be eliminated from the second group, the difference is much more striking, being an average of 11.4 errors for the former and 4.6 for the latter. Thus it is seen that with exceptions the larger combinations are the more difficult, or at least the associations between them and their sums are weaker, than the smaller combinations.

9	6	3	1	2
6	9	2	5	7
17	17	6	7	8
	a	*b*	*c*	*d*

DIAGRAM 14.—Typical errors made in Set A, simple addition

The comparison between Cleveland and Grand Rapids shows certain differences. The errors are more evenly distributed over the 26 combinations for the former than for the latter, the largest number of errors made on any one combination by the Cleveland pupils being 11 as opposed to 17 for the pupils of Grand Rapids. It is also true that, while on the whole there is rather close agreement as to the difficulty of the several combinations, there are several quite marked exceptions to this statement. The 1+5 and 3+2 combinations net more errors in Grand Rapids than in Cleveland, while the reverse is true for 2+7, 9+5, and several others. This would indicate that certain rather freakish associations are established in different groups through peculiar methods of instruction or some other experience common to the individuals making up each of the groups.

This last statement seems to be borne out by the facts presented in Table XIX, in which the fifth and eighth grades are compared. The association 1+5=7, so strongly established in the eighth-grade pupils of Grand Rapids, was found to be relatively weak for the Cleveland group, and is here seen to be quite weak for the fifth grade in Grand Rapids. The association 3+2=6 is strong in both grades. In general the statement made concerning the relative difficulties of the combinations for the eighth grade holds for the fifth.

An additional comment may be made with reference to the formation of wrong associations. An examination of the raw data shows in numerous instances a rather strong persistence of these wrong associations. In addition to those already mentioned there appears another in section *a* of Diagram 14. These two errors, $9+6=17$ and $6+9=17$, were taken from the same record. The repetition of the error would indicate that the association was quite strongly fixed and in all probability denotes a confusion between $9+6$ and $9+8$.

TABLE XIX

A COMPARISON OF THE DISTRIBUTIONS OF 100 ERRORS MADE IN 15 SIMPLE ADDITION COMBINATIONS BY FIFTH- AND EIGHTH-GRADE PUPILS IN GRAND RAPIDS

Grade	Simple Addition Combinations															Total
	1 2	6 6	9 5	0 1	4 2	1 3	7 7	9 6	3 0	2 4	1 5	3 8	6 9	0 7	3 2	
8-2	...	7	6	3	3	1	...	13	...	...	24	10	16	...	17	100
5-2	...	1	10	6	5	4	1	8	3	5	4	11	11	6	25	100
Total	...	8	16	9	8	5	1	21	3	5	28	21	27	6	42	200

SUBTRACTION

The study of errors made in subtraction is confined to Set B. Facts corresponding in essential features to those presented on addition are found in Tables XX and XXI and Diagram 15, bearing on subtraction. The one important difference is that but 20 combinations are studied in the eighth grade and 10 in the fifth.

From the totals in Table XX it is seen that bridging the tens is a relatively much more difficult operation in subtraction than in addition. In the latter it was found that the average number of errors made in the combinations whose sums were greater than ten, excluding certain rather freakish results, was 11.9 as opposed to 4.6 for the combinations whose sums were less than ten. Without making any exception it is found in subtraction that the average number of errors made where the minuend is more than ten is 18.7 (nine cases), while the average where the minuend is ten or less is but 2.9 (eleven cases).

In the comparison of the fifth and eighth grades in Table XXI one very interesting difference is noted. Whereas the eighth-grade

pupils have relatively little difficulty with the combination 1−0, it is by far the most difficult combination for the fifth grade, accounting for 40 errors out of the 100. As will appear in the discussion of multiplication, the pupil either has a great deal of difficulty in getting the conception of zero, or practically no attention is given to it in the course of instruction in arithmetic. From the facts presented in this table it would seem that the understanding of what zero means accompanies the maturing of the pupil. With this one exception differences between the two grades are not particularly evident.

TABLE XX

A Comparison of the Distributions of 100 Errors Made in 20 Simple Subtraction Combinations by Eighth-Grade Pupils in Cleveland and Grand Rapids

City	Simple Subtraction Combinations																				Total
	9 9	7 3	11 6	8 1	12 3	1 0	9 7	13 8	4 3	12 6	8 0	11 9	12 7	5 1	10 2	6 0	11 7	15 8	10 9	12 5	
Cleveland..........	..	2	8	2	14	..	1	11	1	1	1	10	7	1	2	1	18	12	3	5	100
Grand Rapids.......	1	1	4	3	7	5	2	11	..	3	..	5	9	4	..	2	18	20	..	5	100
Total..........	1	3	12	5	21	5	3	22	1	4	1	15	16	5	2	3	36	32	3	10	200

TABLE XXI

A Comparison of the Distributions of 100 Errors Made in 10 Simple Subtraction Combinations by Fifth- and Eighth-Grade Pupils in Grand Rapids

Grade	Simple Subtraction Combinations										Total
	9 9	7 3	11 6	8 1	12 3	1 0	9 7	13 8	4 3	12 6	
8-2...........	2	3	11	8	19	14	5	30	.....	8	100
5-2...........	3	5	8	.....	14	40	5	23	2	.....	100
Total.....	5	8	19	8	33	54	10	53	2	8	200

In Diagram 15 there are presented two typical errors. The error in section *b* of the diagram, 1−0=0, characteristic of the fifth grade, has already been commented upon. The two errors in section *a*, 11−7=5 and 12−7=4, were made by the same pupil.

They indicate the fixing of the wrong associations referred to in connection with the addition combinations. It is evident that the halves of two associations were wrongly paired.

$$\begin{array}{ccccc} 11 & & 12 & \qquad\qquad & 1 \\ \underline{\ 7} & & \underline{\ 7} & & \underline{0} \\ 5 & & 4 & & 0 \\ & a & & & b \end{array}$$

DIAGRAM 15.—Typical errors made in Set B, subtraction

MULTIPLICATION

In studying errors in multiplication two sets were used, C and L. The errors characteristic of these two types will be discussed in their order.

In form, Tables XXII and XXIII are identical with the corresponding tables for addition and subtraction. In the first the eighth grades of Cleveland and Grand Rapids are compared, in the second, the fifth and eighth grades of the latter city. Typical errors, as actually made by the pupils, are reproduced in Diagram 16.

Turning to Table XXII, we note a striking difference between the distribution of errors in multiplication and the distribution of errors in addition and subtraction, already discussed, and in the

TABLE XXII

A COMPARISON OF THE DISTRIBUTIONS OF 100 ERRORS MADE IN 20 SIMPLE MULTIPLICATION COMBINATIONS BY EIGHTH-GRADE PUPILS IN CLEVELAND AND GRAND RAPIDS

City	Simple Multiplication Combinations																				Total
	3 2	4 7	9 8	0 2	5 6	4 1	2 9	7 6	4 0	9 5	9 1	5 2	4 8	7 0	6 5	2 1	3 3	9 6	0 5	7 4	
Cleveland	..	..	..	24	..	..	..	..	26	1	1	..	..	26	..	1	..	2	19	..	100
Grand Rapids	..	2	..	17	..	2	..	..	28	..	..	..	..	28	..	1	2	1	18	1	100
Total	..	2	..	41	..	2	..	..	54	1	1	..	..	54	..	2	2	3	37	1	200

concentration of errors at certain points. It is in those combinations into which zero enters as one of the terms that the largest number of errors is made. This statement is equally true of both

Cleveland and Grand Rapids, as well as of the fifth and eighth grades. The similarity of the results from Cleveland and Grand Rapids is of especial interest and significance when it is remembered that the children of the latter city were familiar with the Courtis tests. Thus, in spite of whatever special training they may have received from these tests on the zero combinations, they registered about the same proportion of errors on these combinations as did the Cleveland children who had not had this special training. Thus it would seem that the handling of the zero is a mental function peculiarly unresponsive to training.

TABLE XXIII

A COMPARISON OF THE DISTRIBUTIONS OF 100 ERRORS MADE IN 10 SIMPLE MULTIPLICATION COMBINATIONS BY FIFTH- AND EIGHTH-GRADE PUPILS IN GRAND RAPIDS

Grade	Simple Multiplication Combinations										Total
	3 2	4 7	9 8	0 2	5 6	4 1	2 9	7 6	4 0	9 5	
8-2	.....	4	.....	35	.....	4	.....	.....	57	.....	100
5-2	1	1	8	25	3	.....	1	3	55	3	100
Total	1	5	8	60	3	4	1	3	112	3	200

A further study of the zero as it enters into the various combinations is interesting. The reader has probably already noticed the greater frequency of errors at 0×4 and 0×7 than at 2×0 and 5×0. Turning now to Diagram 16, sections *a*, *b*, and *c*, we find the three typical performances in dealing with the four zero combinations when an error is made. A pupil may respond correctly to each of the combinations, he may fail on 2×0 and 5×0, he may fail on 0×4 and 0×7, or he may fail on all four combinations. A striking fact suggested by the tables and borne out completely by an examination of the actual work of the children is that the making of errors on these four combinations goes in pairs. That is, there is a tendency to fail on 2×0 and 5×0, while giving the proper reaction to the other two combinations, 0×4 and 0×7, or vice versa. And, as seen in the table, the error is more frequently made in the latter pair of combinations than in the former. Indeed, it

is very rarely the case that a pupil fails on 2×0 and 5×0 while reacting properly to 0×4 and 0×7. Thus it would seem that it is a more difficult mental operation to multiply a quantity by zero than to perform the reverse operation, to multiply zero by the quantity. It is further evident that the two operations are not identical. A very interesting question suggested by all the foregoing is that of the relation between dealing with zero in the simple combinations and dealing with it in the more complex multiplication examples. In section *e* of Diagram 16 we have reproduced the work of a pupil who was quite unable to handle the zero in the

$$\begin{array}{cc} 0 & 4 \\ \underline{2} & \underline{0} \\ 2 & 0 \\ 7 & 0 \\ \underline{0} & \underline{5} \\ 0 & 5 \end{array}\ a \qquad \begin{array}{cc} 0 & 4 \\ \underline{2} & \underline{0} \\ 0 & 4 \\ 7 & 0 \\ \underline{0} & \underline{5} \\ 7 & 0 \end{array}\ b \qquad \begin{array}{cc} 0 & 4 \\ \underline{2} & \underline{0} \\ 2 & 4 \\ 7 & 0 \\ \underline{0} & \underline{5} \\ 7 & 5 \end{array}\ c \qquad \begin{array}{cc} 3 & 7 \\ \underline{3} & \underline{7} \\ 6 & 14 \end{array}\ d$$

$$\begin{array}{ccr} 0 & 4 & 8563 \\ \underline{2} & \underline{0} & \underline{207} \\ 2 & 4 & 59941 \\ & & \underline{17126} \\ 7 & 0 & 1772541 \\ \underline{0} & \underline{5} & \\ 7 & 5 & \end{array}\ e \qquad \begin{array}{ccr} 0 & 4 & 8563 \\ \underline{2} & \underline{0} & \underline{207} \\ 0 & 0 & 59941 \\ & & \underline{171260} \\ 7 & 0 & 231201 \\ \underline{0} & \underline{5} & \\ 0 & 0 & \end{array}\ f$$

DIAGRAM 16.—Typical errors made in multiplication

simple combinations, yet had not the least difficulty with the zero when it appeared in a complex example. In section *f*, on the other hand, there is reproduced the work of a pupil who had difficulty in the reverse order, handling all the zero combinations correctly, but failing on the zero in the complex example. It should also be said that in not a single case out of 50 examined did the writer find difficulty with the former to be correlated with difficulty with the latter.

One more type of error should be mentioned before we leave multiplication combinations, and that is the error reproduced in section *d* of the diagram, which is mentioned earlier in the chapter in connection with the discussion of addition. The two errors reproduced, made by the same individual, are $3\times 3=6$ and $7\times 7=14$.

These are the typical errors made on these combinations. When we consider that these responses were made to the combinations when they appeared in the midst of a set of multiplication examples, when the pupils were actually in the multiplying attitude, the evidence points to a relatively strong additional association with these combinations. Whether or not there is a tendency to add like quantities is a problem demanding further experimental evidence for its solution.

Let us proceed now to the study of the errors made in the examples of Set L, the multiplication of four-place by two-place numbers. The facts, as they have been secured, appear in Table XXIV. In this table is found the distribution of 100 errors made by the eighth-grade pupils. It will be noted that the errors are thrown into three categories, "mistakes in multiplying," "mistakes in adding," and "other mistakes." It is to be regretted that a more detailed study might not have been made of the errors in these examples, but this was found to be impossible from the mere records of the pupils. It would have been desirable to isolate errors caused through "carrying" in performing the multiplication and "carrying" in performing the addition necessary to the solution of this type of example, but it was impossible to determine whether a mistake that appeared in one of the partial products was due to difficulty with the tables or with "carrying." Thus "mistakes in multiplying" is a composite including both mistakes in the tables and mistakes in "carrying," as well as mistakes due to the combination of these two operations. "Mistakes in adding" is likewise a composite. "Other mistakes" includes mistakes due to crowding of figures, difficulty with the mechanics of multiplication, etc.

TABLE XXIV

DISTRIBUTION OF 100 ERRORS MADE IN COMPLEX MULTIPLICATION EXAMPLES, SET L—EIGHTH GRADE

Mistakes in multiplying	72
Mistakes in adding	25
Other mistakes	3
Total	100

A word concerning the table itself will suffice. Mistakes in multiplying are by far the most frequent. This merely confirms facts brought out in the tables on accuracy which showed that addition is always performed with relatively greater accuracy than multiplication.

DIVISION

In the study of typical errors in division, the study is limited to Sets D and N; Set I is eliminated because of the impossibility of isolating mistakes; and the facts for Set K are not presented because it is of the same general character as Set N.

The results of the study of Set D are found in Tables XXV and XXVI and Diagram 17. In essential respects the tables are like the tables for the three other sets of the simple combinations and

TABLE XXV

A COMPARISON OF THE DISTRIBUTIONS OF 100 ERRORS MADE IN 21 SIMPLE DIVISION COMBINATIONS BY EIGHTH-GRADE PUPILS IN CLEVELAND AND GRAND RAPIDS

City	Simple Division Combinations																					Total
	$\frac{9}{3}$	$\frac{32}{4}$	$\frac{36}{6}$	$\frac{0}{2}$	$\frac{28}{7}$	$\frac{9}{9}$	$\frac{21}{3}$	$\frac{48}{6}$	$\frac{1}{1}$	$\frac{10}{5}$	$\frac{6}{2}$	$\frac{24}{4}$	$\frac{63}{7}$	$\frac{0}{6}$	$\frac{32}{8}$	$\frac{8}{1}$	$\frac{30}{5}$	$\frac{72}{8}$	$\frac{0}{1}$	$\frac{36}{9}$	$\frac{7}{1}$	
Cleveland	..	1	4	2	..	32	1	2	34	..	1	1	2	5	1	2	..	2	3	2	5	100
Grand Rapids	..	3	2	1	2	26	..	5	24	3	..	8	2	1	1	4	..	5	3	1	9	100
Total	..	4	6	3	2	58	1	7	58	3	1	9	4	6	2	6	..	7	6	3	14	200

therefore they require no explanation. Attention is directed, however, to the fact that the results for 21 combinations instead of for 20 are presented in the first table.

An examination of the tables shows that the most frequent error is made in those examples in which a quantity is divided by itself.

$$1\overline{)1} \qquad 9\overline{)9}$$
$$0 \qquad\quad 0$$

DIAGRAM 17.—Typical error made in Set D, division

The nature of this error is made clear by reference to Diagram 17. Here it is seen that there is a tendency for the child to say $1 \div 1 = 0$, $9 \div 9 = 0$. This is the typical error made in the simple division com-

binations. It is difficult for the child to know that there is anything left when a quantity is divided by itself. It is at this point that division and subtraction become confused.

The records from Cleveland and Grand Rapids are in distinct agreement in showing this to be the most frequent error. Relatively, however, it seems to be made more frequently by children from the former than by those from the latter city. Table XXVI shows this error to stand out prominently in the fifth grade as well as in the eighth. The fifth-grade pupils, however, have relatively more difficulty with other combinations than do the eighth-grade pupils. This is just what should be expected when we consider that it merely indicates a more complete mastery of the tables on the part of the pupils in the more advanced grade.

TABLE XXVI

A COMPARISON OF THE DISTRIBUTIONS OF 100 ERRORS MADE IN 10 SIMPLE DIVISION COMBINATIONS BY FIFTH- AND EIGHTH-GRADE PUPILS IN GRAND RAPIDS

Grade	Simple Division Combinations										Total
	$\frac{9}{3}$	$\frac{32}{4}$	$\frac{36}{6}$	$\frac{0}{2}$	$\frac{28}{7}$	$\frac{9}{9}$	$\frac{21}{3}$	$\frac{48}{6}$	$\frac{1}{1}$	$\frac{10}{5}$	
8-2	.....	5	3	2	3	39	.....	8	36	4	100
5-2	4	5	13	4	5	25	6	7	26	5	100
Total	4	10	16	6	8	64	6	15	62	9	200

In studying the errors made in examples in long division, Set N, they were grouped into three groups corresponding to the three

TABLE XXVII

DISTRIBUTION OF 100 ERRORS MADE IN EXAMPLES IN LONG DIVISION, SET N—EIGHTH-GRADE PUPILS

Mistakes in multiplying	68
Mistakes in subtraction	26
Other mistakes	6
Total	100

groups employed in connection with the multiplication, Set L. These three groups, as shown in Table XXVII, are "mistakes in

multiplying," "mistakes in subtraction," and "other mistakes." An explanation of these terms is not necessary. Suffice it to say that each of the groups of errors has the same composite character as was referred to in the discussion of multiplication.

The table shows marked similarity to Table XXIV. A large number of mistakes is made in multiplying, and in the processes incident thereto, while a comparatively small number is made in subtraction. This further bears out the proposition that multiplication is a relatively difficult mental operation.

FRACTIONS

Fractions, it will be remembered, are represented in the test by two sets, H and O. In the first, fractions of like denominators are added and subtracted; in the second, fractions of unlike denominator are added, subtracted, multiplied, and divided. As compared with the more complex examples in the "fundamentals," it has been found relatively easy to make a study of errors. From the result set down by the pupil it is possible in most cases to determine what he did and how he did it. For this reason a more exhaustive study has been undertaken of the errors made in these two sets than was possible in connection with any one of the four fundamental operations. It should be added further that, since each of these sets is a complex, a separate study has been made of the errors occurring in each of the types of operation found in the sets. Thus the study of Set H is divided into two parts, the one concerned with the addition, the other with the subtraction, of fractions of like denominators. The study of Set O is consequently divided into four parts, dealing respectively with addition, subtraction, multiplication, and division of fractions of unlike denominators.

FRACTIONS OF LIKE DENOMINATORS

One hundred errors made in the addition of fractions of like denominators by eighth-grade pupils were analyzed and thrown into the five categories which appear in Table XXVIII. In dealing with these simple fractions the most frequent error is found to be that indicated by the reproduction in section *a*, Diagram 18. The numerators are added and likewise the denominators. This is

perhaps the mistake that would be expected, and it should therefore be very carefully guarded against by the teacher. It shows, however, that the child does not have the least conception of the meaning of denominator.

TABLE XXVIII

Frequency of Types of Error in Addition of Fractions of Like Denominators—Eighth Grade

Type of Error	Frequency
Numerators added, denominators added	60
Numerators multiplied, denominators multiplied	27
Numerators added, denominators multiplied	8
Common denominator found	4
Numerators multiplied, denominators added	1
Total	100

Another frequent error is that shown in section *b* of the diagram; the numerators are multiplied and likewise the denominators. This indicates an interference of mental functions. Since the pupil

$$\frac{3}{5}+\frac{1}{5}=\frac{4}{10} \qquad \frac{1}{9}+\frac{5}{9}=\frac{6}{18}$$

a

$$\frac{3}{5}+\frac{1}{5}=\frac{3}{25} \qquad \frac{1}{9}+\frac{5}{9}=\frac{5}{81}$$

b

$$\frac{3}{5}+\frac{1}{5}=\frac{4}{25} \qquad \frac{1}{9}+\frac{5}{9}=\frac{6}{81}$$

c

$$\frac{3}{5}+\frac{1}{5}=\frac{20}{25} \qquad \frac{1}{9}+\frac{5}{9}=\frac{54}{81}$$

d

$$\frac{3}{5}+\frac{1}{5}=\frac{3}{10} \qquad \frac{1}{9}+\frac{5}{9}=\frac{5}{18}$$

e

$$\frac{3}{5}+\frac{1}{5}=\frac{4}{10} \qquad \frac{11}{15}+\frac{1}{6}=\frac{27}{30}$$

f

Diagram 18.—Typical errors made in adding fractions of like denominators

is an eighth-grade pupil, the multiplication of fractions is more vividly in his mind than is addition. He consequently multiplies. Sometimes an individual will be found who multiplies everything

in Sets H and O. This may be due to carelessness in observing signs; or it may be due to the fixing of the method of handling one type of fractions at the expense of others.

Another type of error is found in section *c* of the diagram. The numerators are added and the denominators multiplied. This indicates a confusion between the method of adding and the method of multipling fractions. A modification of this same type of confusion appears in section *e* where the numerators are multiplied and the denominators added. And finally in section *d* we have a performance which, while not an error, strictly speaking, indicates a slavish adherence to the mechanics of fractions. The pupil, instead of simply adding the numerators, first found a common denominator and then added the new numerators. The result is not wrong, but the method used to get it is wrong.

In the last section of the diagram, *f*, the work of a pupil in adding fractions of like denominators and in adding fractions of unlike denominators is reproduced. It would seem that if a pupil could work examples of the more complex type he would have no difficulty in working those of the simple type, especially when it is borne in mind that in the course of instruction he encounters the latter before the former, and it is supposed by all that mastery of the simple necessarily underlies, and leads up to, the more complex operation. From this section of the diagram it is evident that this is a false assumption. Unless the pupil is given an understanding of the nature of fractions, he becomes a slave to the method; and the learning of the method of handling one type of fractions seems in no way to involve the learning of the method of handling a simpler type of fractions, although the understanding of the former does involve the understanding of the latter.

In Table XXIX and Diagram 19 there appears a corresponding analysis of the types of error made by eighth-grade pupils in the subtracting of fractions of like denominators. It will be noted that the most frequent error is "confusion of symbols," while this error did not occur at all in the addition of similar fractions. This is due to an unhappy organization of the set. An examination of this set shows it to be composed of four columns of examples. All those in the first column are to be added, all in the second sub-

tracted, all in the third added, and all in the fourth subtracted. Now, since the fractions in the first column are to be added, the pupil quite frequently obeys the suggestion that all the fractions in the set are to be added. Thus "confusion of symbols" appears as a frequent type of error in subtraction and is absent from addition.

TABLE XXIX

FREQUENCY OF TYPES OF ERROR IN SUBTRACTION OF FRACTIONS OF LIKE DENOMINATORS—EIGHTH GRADE

Type of Error	Frequency
Confusion of symbols	43
Numerators subtracted, denominators subtracted	25
Numerators multiplied, denominators multiplied	23
Numerators added, denominators multiplied	6
Numerators subtracted, denominators added	3
Total	100

The next most frequent error corresponds to the most frequent error in addition, the numerators being subtracted and the denominators subtracted. The exact character of this error is made clear through section *a* of Diagram 19. Another error appearing with about the same frequency in subtraction as in addition is the multiplication of both numerators and denominators. A third mistake,

a	*b*	*c*	*d*
$\frac{6}{9}-\frac{4}{9}=\frac{2}{0}$	$\frac{6}{9}-\frac{4}{9}=\frac{24}{81}$	$\frac{6}{9}-\frac{4}{9}=\frac{10}{81}$	$\frac{6}{9}-\frac{4}{9}=\frac{2}{18}$
$\frac{3}{7}-\frac{1}{7}=\frac{2}{0}$	$\frac{3}{7}-\frac{1}{7}=\frac{3}{49}$	$\frac{3}{7}-\frac{1}{7}=\frac{4}{49}$	$\frac{3}{7}-\frac{1}{7}=\frac{2}{14}$

DIAGRAM 19.—Typical errors made in subtracting fractions of like denominators

the exact nature of which is perhaps due to confusion of signs, but which would be an error even though the signs were changed, is reproduced in section *c* of the diagram. The numerators are added and the denominators multiplied. A fourth error, which is difficult to explain, appears in section *d*, the numerators being subtracted and the denominators added. It indicates, however, a complete

dependence upon method or formulae, divorced from an understanding of the operation to be performed.

FRACTIONS OF UNLIKE DENOMINATORS

In Table XXX the accuracy, or rather the degree of error, with which each of the examples in Set O is worked by the eighth-grade pupils in Cleveland and Grand Rapids is indicated. The facts in

TABLE XXX

NUMBER OF PUPILS OUT OF 100 FAILING ON EACH EXAMPLE IN SET O. EIGHTH GRADES OF CLEVELAND AND GRAND RAPIDS COMPARED

City	Examples in Fractions												Total
	[illegible] + [illegible]	[illegible] + [illegible]	[illegible] + [illegible]	$\frac{9}{14}-\frac{1}{4}$	[illegible] − [illegible]	[illegible] − [illegible]	[illegible] × [illegible]	[illegible] × [illegible]	[illegible] × [illegible]	[illegible] ÷ [illegible]	[illegible] ÷ [illegible]	[illegible] ÷ [illegible]	
Cleveland......	40	40	49	51	48	40	13	11	10	49	37	44	432
Grand Rapids..	44	51	53	50	46	44	13	14	17	39	38	36	445
Total......	84	90	102	101	94	84	26	25	27	88	75	80	877

this table were secured as follows: From the records of the eighth-grade pupils of each of the cities there were taken at random the records of 100 pupils who had attempted all twelve examples. It may be argued that such a method necessarily involves a selection of either a superior or an inferior set of records. That such, however, is not the case to any marked degree is shown by a comparison of the percentage of accuracy for the Cleveland and Grand Rapids eighth grades as represented in Diagram 11, with the corresponding measures of accuracy for the records presented in Table XXX. From Diagram 11 we find the percentages of accuracy for Cleveland and Grand Rapids to be 67.6 and 66.0, respectively, while, as computed from the table, the corresponding percentages are 64.0 and 62.9. It is probable, however, that, even though a superior or inferior group were selected, the distribution of errors would not be far from valid.

The table shows great variability of difficulty among the 12 examples. This is seen at once to be due to the composite character of the set. The three examples representing each type of operation show quite close agreement in the number of errors made on each.

Likewise the differences between the two cities are not great. From this table the average percentage of error has been determined for each of the four types of fractions, viz., addition, subtraction, multiplication, and division. A graphical representation of these facts appears in Diagram 20. From this diagram it is seen that subtraction is the most difficult operation, followed in order by addition, division, and multiplication; the last-named operation is found to be especially easy, while there are no great differences among the other three.

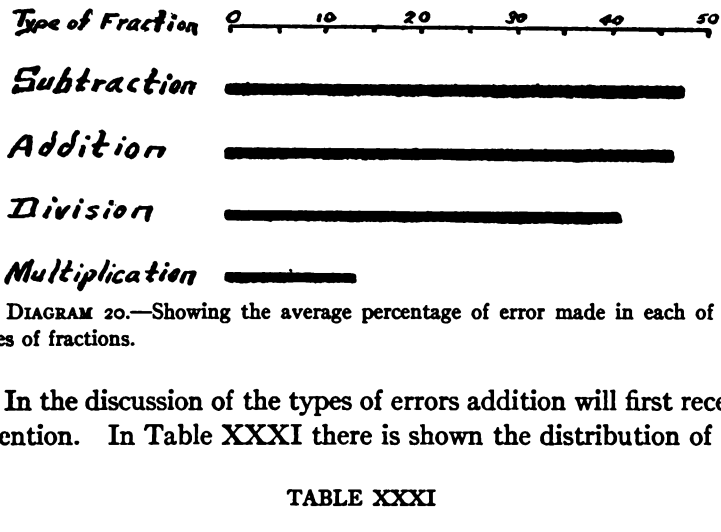


DIAGRAM 20.—Showing the average percentage of error made in each of four types of fractions.

In the discussion of the types of errors addition will first receive attention. In Table XXXI there is shown the distribution of 100

TABLE XXXI

FREQUENCY OF TYPES OF ERROR IN ADDITION OF FRACTIONS OF UNLIKE DENOMINATORS—EIGHTH GRADE

Type of Error	Frequency		
	Cleveland	Grand Rapids	Total
Numerators added, denominators added	30	46	76
Numerators multiplied, denominators multiplied	22	22	44
Mistakes in "fundamentals"	35	10	45
Confusion of symbols	2	10	12
Numerators multiplied, denominators added	4	8	12
Numerators added, denominators multiplied	2	2	4
Method obscure	5	2	7
Total	100	100	200

errors for both Cleveland and Grand Rapids eighth-grade pupils. In Diagram 21 there are five typical errors as the work was actually done by the pupils.

$\frac{11}{15}+\frac{1}{6}=\frac{12}{21}=\frac{4}{7}$ *a* $\quad \frac{11}{15}+\frac{1}{6}=\frac{11}{90}$ *b* $\quad \frac{11}{15}+\frac{1}{6}=\frac{11}{21}$ *c* $\quad \frac{11}{15}+\frac{1}{6}=\frac{12}{90}$ *d* $\quad \frac{11}{15}+\frac{1}{6}=\frac{15}{66}$ *e*

DIAGRAM 21.—Typical errors made in adding fractions of unlike denominators

The most frequent error is the one found to occur most often in the addition of fractions of like denominators, viz., the addition of both numerators and denominators (see section *a* of the diagram). Each of the other five types of error, with the exception of "mistakes in fundamentals" and "confusion of symbols," which are easily understood, is reproduced in sections *b*, *c*, and *d* of the diagram. These three errors are also the typical errors found in the addition of fractions in Set H. There were several errors found which it was impossible to analyze. These were all grouped under "method obscure." An error which was very rare, but interesting, appears in section *e* of the diagram. This pupil inverted the dividend and multiplied. It was thus an incorrect method for division, although that was undoubtedly the method influencing the pupil to make a response of that sort.

A comparison of the records made by the Cleveland pupils and the Grand Rapids pupils shows some rather clear differences. The Cleveland pupils seem to be especially weak on the "fundamentals" when they are used in connection with the solving of fractions. That is, they have the proper method of dealing with the fractions, but make mistakes in the simple combinations. The Grand Rapids children, on the other hand, have a relatively stronger predilection to add both numerators and denominators than do the Cleveland children. In other respects the two groups are not greatly different.

Turning now to Table XXXII and Diagram 22, we find the facts presented for the subtraction of fractions of unlike denominators. The table shows the most frequent error to be the subtraction of both numerators and both denominators. It will be noticed that this error has about the same frequency as the corresponding error in addition, viz., the addition of both numerators and of both

denominators. This error may be of two kinds, the most frequent being represented in section *a* of Diagram 22. In the other, the subtraction is in the same direction for both numerator and denominator. Thus $5-2$ would give 3 for the numerator and $6-21$ would give 0 for the denominator, instead of 15, obtained by subtracting the other way, $21-6$, as in section *a*.

TABLE XXXII

FREQUENCY OF TYPES OF ERROR IN SUBTRACTION OF FRACTIONS OF UNLIKE DENOMINATORS—EIGHTH GRADE

TYPE OF ERROR	FREQUENCY		
	Cleveland	Grand Rapids	Total
Numerators subtracted, denominators subtracted...	20	59	79
Confusion of symbols....	37	16	53
Mistakes in fundamentals....	24	5	29
Numerators added, denominators multiplied....	4	4	8
Numerators added, denominators added....	4	2	6
Numerators subtracted, denominators multiplied....	2	2	4
Miscellaneous....	3	7	10
Method obscure....	6	5	11
Total....	100	100	200

The confusion of symbols is more frequent here than in addition. This difference will be explained in comparing Cleveland and Grand Rapids results. The other types of errors, "numerators added and denominators multiplied," "numerators added and denominators

$\frac{5}{6}-\frac{2}{21}=\frac{3}{15}=\frac{1}{5}$ (*a*) $\quad \frac{5}{6}-\frac{2}{21}=\frac{7}{126}$ (*b*) $\quad \frac{5}{6}-\frac{2}{21}=\frac{7}{27}$ (*c*) $\quad \frac{5}{6}-\frac{2}{21}=\frac{3}{126}$ (*d*) $\quad \frac{5}{6}-\frac{2}{21}=\frac{12}{105}$ (*e*)

DIAGRAM 22.—Typical errors made in subtracting fractions of unlike denominators

added," and "numerators subtracted and denominators multiplied," are shown in sections *b*, *c*, and *d* of the diagram and therefore need not be discussed. To include certain other errors, very rarely made, the category "miscellaneous" has been introduced into the table. In the last section of the diagram, section *e*, there is reproduced an error made by the same pupil who made the error shown in the same section in Diagram 21. The error is also the same, the

inversion of one of the fractions, followed by multiplication. This incorrect method of dividing seems to be quite strongly fixed in this individual.

In comparing the records made by the Cleveland and Grand Rapids children, the differences seem to be greater than the similarities in the three types of errors occurring most frequently. The difference between the frequency of "confusion of symbols" in the two cases is explained by a difference in the organization of Set O as the test was given to the two cities. When given to Cleveland the set was composed of four columns of examples, 3 to each column, and the 3 examples in each column were of the same type. The examples in the first column were addition, those in the second subtraction, those in the third multiplication, and those in the fourth division. Because of this arrangement the principle of suggestion operated in this set as it did in Set H, previously mentioned, and caused a greater frequency of confusion of symbols for Cleveland than for Grand Rapids. The greater frequency of mistakes in fundamentals, the simple combinations, among the Cleveland children has been commented on in connection with the examples in addition. The Cleveland children likewise seem to be less inclined to subtract than to add both numerators and both denominators.

Table XXXIII and Diagram 23, corresponding to the tables and diagrams for addition and subtraction, indicate the character of the errors made by the eighth-grade pupils in the multiplication of fractions. The most frequent error in performing this operation is shown in section *a* of the diagram. In this case the pupil first finds the least common denominator and then adds the resulting numerators. This is really the method used for adding fractions, and a large portion of these errors can in all probability be accounted for by confusion of symbols. The next most frequent error, however (section *b*, Diagram 23), represents a confusion of the method of addition and that of multiplication. The pupil begins with the former method and ends with the latter. Thus he finds the least common denominator and then multiplies the resulting numerators, leaving the least common denominator as it is and setting it down as the denominator of the fractional product. The other mistake which is made in connection with the least common denominator

idea (section *g*, Diagram 23), though much less frequently encountered, may be profitably discussed here. The least common denominator is found, and then the resulting numerators are multiplied and the least common denominator squared to give the numerator and denominator of the product. It will be noted that the result secured in this way is not incorrect, but the method is bad. From these three types of errors it is evident that the idea of the least common denominator interferes with, and highly complicates, the very simple method of multiplying fractions.

TABLE XXXIII

FREQUENCY OF TYPES OF ERROR IN MULTIPLICATION OF FRACTIONS OF UNLIKE DENOMINATORS—EIGHTH GRADE

TYPE OF ERROR	FREQUENCY		
	Cleveland	Grand Rapids	Total
L. C. D., numerators added	36	25	61
L. C. D., numerators multiplied	28	15	43
Mistakes in fundamentals	12	12	24
Numerators added, denominators added	8	4	12
Numerators added, denominators multiplied	3	8	11
Inversion, numerators multiplied, denominators multiplied	....	13	13
Inversion of results	....	10	10
L. C. D., numerators multiplied, denominators multiplied	2	....	2
Miscellaneous	1	8	9
Method obscure	10	5	15
Total	100	100	200

The remaining types of errors, represented by sections *c*, *d*, *e*, and *f* of the diagram, require some comment at this point. In the first of these, *c*, we have the addition of both numerators and both denominators again, in *d* the addition of the numerators and the multiplication of denominators, in *e* the application of the method of division, and in *f* the inversion of results. The last-named error is one of that peculiar type already represented by difficulty in the conception of zero and of unity noted in connection with the simple multiplication and division combinations. The table shows this error to be comparatively frequent in Grand Rapids, while it is not represented by a single instance in the records studied for

Cleveland. This same mistake occurs in the first part of section *h* of the diagram. It is difficult for a pupil to get the distinction between $\frac{20}{1}$ and $\frac{1}{20}$, or it may rather be that the pupil tends to discard the 1 when it appears as the numerator just as it is commonly discarded when it appears as the denominator. This represents a very interesting type of confusion.

$$\frac{1}{6}\times\frac{3}{10}=\frac{14}{30} \qquad \underset{b}{\frac{1}{6}\times\frac{3}{10}=\frac{45}{30}} \qquad \underset{c}{\frac{1}{6}\times\frac{3}{10}=\frac{4}{16}} \qquad \underset{d}{\frac{1}{6}\times\frac{3}{10}=\frac{4}{60}}$$

$$\underset{e}{\frac{1}{6}\times\frac{3}{10}=\frac{10}{18}} \qquad \underset{f}{\frac{1}{6}\times\frac{3}{10}=20} \qquad \underset{g}{\frac{1}{6}\times\frac{3}{10}=\frac{45}{900}}$$

$$\frac{\cancel{2}}{\cancel{5}}\times\frac{\cancel{5}}{\underset{2}{\cancel{6}}}=2$$

$$\frac{\cancel{5}}{6}\times\frac{19}{\underset{4}{\cancel{20}}}=\frac{19}{6}$$

$$\underset{h}{\frac{1}{\underset{2}{\cancel{6}}}\times\frac{\cancel{3}}{10}=\frac{1}{10}}$$

DIAGRAM 23.—Typical errors made in multiplying fractions of unlike denominators

In this same section (*h*) we find another peculiar type of error. The work on all these examples was done by the same pupil. The pupil has learned to cancel, but not to use the results of cancellation, except in the first of the examples, where it seems that he has used the "2" because of inability to find anything else to put down as a result.

In comparing Cleveland and Grand Rapids certain differences are noted. The children of the former seem to be more inclined to find a least common denominator than do the children of the latter city. This is partially accounted for by the difference in the organization of the test when given to the two groups, already referred to. In the inversions of terms and of results Grand Rapids monopolizes all the errors found. In other respects the records from the two cities are not greatly different.

Turning now to Table XXXIV and Diagram 24, we find the facts presented for the last type of examples found in Set O, division. Failure to invert the divisor (section *a*, Diagram 24) seems to be the most frequent error. Another interesting and logical error is the dividing of the numerator of one of the fractions by the numerator of the other and the denominator of one by the denominator of the other (section *b*, Diagram 24). The rule is that the larger quantity is divided by the smaller rather than the term of the dividend by the term of the divisor. A very interesting example

TABLE XXXIV

Frequency of Types of Error in Division of Fractions of Unlike Denominators—Eighth Grade

Type of Error	Frequency		
	Cleveland	Grand Rapids	Total
Failure to invert	33	26	59
Mistakes in fundamentals	27	20	47
Numerators divided, denominators divided	2	20	22
L. C. D., numerators added	14	6	20
Numerators added, denominators added	3	4	7
Inversion of dividend	2	5	7
L. C. D., numerators subtracted	3	3	6
Miscellaneous	2	4	6
Method obscure	14	12	26
Total	100	100	200

of this error is found in section *g* of the diagram. Here the pupil has introduced decimals into the operation. Another error deserving mention is that reproduced in section *e*. The dividend is inverted instead of the divisor. The other types of errors have all appeared in connection with the other types of examples and have been discussed; hence nothing further need be said regarding them.

There also seem to be some differences between Cleveland and Grand Rapids in the division of fractions. Children of the former city fail to invert; that is, they employ the method of multiplication more frequently than do children of the latter city. This is probably due in a measure to the difference in the organization of

the test as given in the two cities. Cleveland children show greater weakness in fundamentals here as in the other operations. The addition suggestion already discussed is found to operate on division for the Cleveland children. The Grand Rapids children, on the other hand, seem prone to divide the one numerator by the other and the one denominator by the other. These statements cover the chief differences.

$$\frac{20}{21} \div \frac{1}{6} = \frac{20}{126} \quad (a) \qquad \frac{20}{21} \div \frac{1}{6} = \frac{20}{3\frac{3}{6}} \quad (b) \qquad \frac{20}{21} \div \frac{1}{6} = \frac{47}{42} \quad (c)$$

$$\frac{20}{21} \div \frac{1}{6} = \frac{21}{27} \quad (d) \qquad \frac{20}{21} \div \frac{1}{6} = \frac{21}{120} \quad (e) \qquad \frac{20}{21} \div \frac{1}{6} = \frac{33}{42} \quad (f)$$

$$\frac{11}{12} \div \frac{5}{8} = \frac{2.6}{1.4}$$

$$\frac{5}{6} \div \frac{11}{15} = \frac{2.1}{2.3}$$

$$\frac{20}{21} \div \frac{1}{6} = \frac{20}{3.3} \quad (g)$$

DIAGRAM 24.—Typical errors made in dividing fractions of unlike denominators

SUMMARY

1. In the addition of the simple combinations the general proposition seems to be established that on the average those combinations whose sums exceed ten are more difficult than those whose sums are less than ten. To this general statement there are individual exceptions which indicate the formation of peculiarly strong associations, some being right and others wrong. These peculiar associations vary among different groups. This would indicate that the formation of the association is to be accounted for in terms of the experience of the group rather than in the character of the combination itself.

2. In the simple subtraction combinations "bridging the tens" is found to be a relatively much more difficult operation than in the addition combinations. Freakish errors, on the other hand, are found to be less frequent in the former than in the latter. The

understanding of the meaning of zero seems to accompany the maturing of the pupil. This is indicated by a relatively large percentage of errors made on the combination 1 − 0 by fifth-grade pupils, whereas this combination presented but little difficulty to pupils in the eighth grade.

3. Practically all the errors made in the simple multiplication combinations are made in those combinations in which zero enters as one of the terms. Furthermore, it is a more difficult mental operation to multiply a quantity by zero than to perform the reverse operation, to multiply zero by the quantity. And a pupil may have difficulty with the zero in the simple combinations, yet be quite able to handle it in the more complex examples, and vice versa. In the complex multiplication examples the most frequent error is made in multiplying.

4. In the simple division combinations the most frequent error is made in dividing a quantity by itself. The result given is zero, showing a confusion between the division and subtraction processes. In long division the demand for multiplication accounts for most of the errors.

5. The typical errors made in working fractions indicate, as a general rule, a slavish adherence to the mechanics of fractions and show emphasis upon method rather than upon an understanding of the process. There consequently follows a great deal of confusion of methods on the part of the pupil.

6. In the addition and subtraction of fractions of like denominator there is a tendency to add both numerators and denominators in the one case and subtract them in the other.

7. In the working of fractions of unlike denominator those involving subtraction are found to be the most difficult, followed in order of decreasing difficulty by those involving addition, division, and multiplication. Multiplication of such fractions is shown to be especially easy.

8. In the application of each of the fundamental operations to fractions there seem to be certain types of errors which recur again and again. Careful attention on the part of the teacher to these typical errors would be worth while.

CHAPTER V

A COMPARISON OF THE ARITHMETICAL ABILITIES OF CERTAIN AGE AND PROMOTION GROUPS

One of the great values of a standard test is to throw light on our methods of instruction, the general organization of our courses of study, our system of promotion, and so on, through an analysis of what children do under these different influences. The present study represents an attempt to compare the arithmetical abilities or attainments of certain age and promotion groups. It therefore falls into two divisions, closely related and dealing largely with the same problem, the one concerning itself with differences in groups of children classified according to age, and the other with differences in groups classified according to rates and causes of promotion or non-promotion. Although these two divisions of the study are very closely related, they will be treated separately for the sake of convenience.

AGE GROUPS

The purpose of this division of the investigation is to find out whether or not there are any differences in the arithmetical abilities which accompany differences in the age of pupils in the same grade, that is, whether the under-age group is at all different from the over-age group, or whether the intermediate or normal group differs from either of these. Of course the test employed is quite inadequate to indicate all differences in arithmetical abilities, but in so far as the test is adequate the nature of the differences, if any exist, will be analyzed.

METHOD

The data upon which this part of the study is based were secured from the results of the arithmetic test given to the children in the B sections of Grades 3–8 inclusive of the Cleveland schools. The giving of the test has already been discussed and therefore need not be taken up here.

On the first page of each folder the age of the pupil taking the test was called for, as indicated in the reproduction of the test in chapter ii, except that it called for age in years only, not in years and months as in the revised test. Thus we had recorded the age of each of the children taking the test. It was therefore possible to group the pupils in each grade according to age.

Now it will be remembered that, while the median results for Cleveland as a whole seemed to be devoid of any considerable inaccuracy, there was some doubt as to the accuracy of any particular record, owing to the fact that the test is a complicated one involving time allowances difficult to administer exactly, and to the fact that it was given by teachers with little or no training in giving tests of this sort. Thus it is evident that, if our comparisons are to be valid, some method must be adopted which will eliminate those errors which may have been made in timing.

Furthermore, an examination of the results of any standard test secured for the various schools and classes of a large city system shows that there are large differences from school to school and from class to class that are to be accounted for by differences in the training which the pupils in the different schools and classes have received. This must also be taken care of by our method; otherwise differences between two age groups might be due to differences in training rather than to differences in age.

Thus it is seen that there are two factors which might account for differences between two groups that must be eliminated. The first of these is differences in giving the test. Overtiming or undertiming would favor or prejudice one group with reference to another. The second of these is differences in training. One group may show superiority over another because its members have had a more effective course of training. In order, therefore, that any comparisons which are made may be valid, it is necessary that the groups compared be homogeneous as to the conditions under which the test was given and as to training. Of course there are other minor factors which may have influence, but it is believed by the writer that if these two are taken care of the comparisons will be valid.

After an examination of the records had showed that it would be possible to secure data on four age groups, the records made by

the pupils of a grade in a particular school were divided into two groups on the basis of sex; then each of these was thrown into four groups on the basis of age. Since, however, the age of each pupil was given in years only, it was impossible to divide the boys and girls of an entire class into four equal groups. For instance, suppose we have a third-grade class of 20 children. It is probable that 10 of these will be boys and 10 girls. Of the 10 boys it is probable that 1 will be seven years old, 4 eight years old, 4 nine years old, and 1 ten years old or more. Now in order that each of our four age groups may be equally influenced by the giving of the test to this class and by the training which the class has received, it is necessary that this class be equally represented in each group. Since there is but one pupil in the lower age group and but one in the upper, one must be taken at random from each of the intermediate groups. The same method is followed for the girls. Thus from this class of 20 pupils but 4 boys and 4 girls have been taken, because the ages were given in years.

This method of selecting pupils for each of the age groups was continued until there were secured records from 50 boys and 50 girls for each of the age groups in each grade from the third to the eighth inclusive. This made a total of 100 records for each group in each grade, or 400 records for each grade, making a grand total of 2,400 records, upon which this study is based. In order to secure this number, the records made by 40–50 schools were analyzed for each grade. And it should be reasserted that the four age groups in each of the grades (100 pupils to the group) represent experience in taking the test as nearly identical as it is possible to make it, and also, after allowing for differences due to transfer from one school to another, identical training so far as training in the school is concerned.

The facts concerning the ages of these groups in the several grades are given in Table XXXV. Here the average age of each group is given. And it should be added that the range of the ages in any one group is practically confined to two years except in the case of Group IV, in which the range is about three years. This means that while the ages of the pupils of one group are not identical, because of differences in the same grade from school to school

in this respect, the groups are quite homogeneous as to age. The table shows that the difference between the average age of each group and the average of the next older group in each grade is at least a year in every instance, and in some cases it is considerably more. Thus the groups are seen to represent real age differences.

TABLE XXXV

AVERAGE AGE OF EACH OF FOUR AGE GROUPS IN GRADES 3–8. 50 BOYS AND 50 GIRLS PER GROUP IN EACH GRADE

Grade	Group			
	I	II	III	IV
3........	7.7	8.8	9.9	11.6
4........	8.7	9.8	10.9	12.7
5........	9.7	10.7	12.0	13.6
6........	10.5	11.6	12.9	14.4
7........	11.4	12.5	13.7	15.0
8........	12.1	13.1	14.3	15.5

After these records had been secured they were all carefully regraded, lest any error due to the scoring of the pupils should prejudice the results. They were then tabulated, both the number of examples attempted and the number of examples worked correctly in each of the sets of the test. And, finally, average "rights," average "attempts," and accuracy were determined for each age group in each grade for each of the sets.

RESULTS

The detailed facts concerning the number of examples worked correctly by the four groups in the six grades appear in Table XXXVI. In order that the table may be made perfectly clear to the reader an explanation is necessary. The Roman numerals, I, II, III, and IV, represent the four age groups in each grade. Group I is the under-age group, Groups II and III are the intermediate or normal groups, and Group IV is the over-age group. Keeping this explanation in mind, we read that in the third grade Group I, the under-age group, worked correctly on the average 16.6 examples in Set A, 10.7 in Set B, and so on. Group II, the younger

of the two intermediate groups, averaged 15.9 examples correctly worked in Set A, 9.1 in Set B, and so on. In this same way the

TABLE XXXVI

AVERAGE "RIGHTS" IN EACH SET FOR EACH OF FOUR AGE GROUPS IN GRADES 3-8. DATA FROM 1400 PUPILS

Set	Third Grade I	Third Grade II	Third Grade III	Third Grade IV	Fourth Grade I	Fourth Grade II	Fourth Grade III	Fourth Grade IV	Fifth Grade I	Fifth Grade II	Fifth Grade III	Fifth Grade IV
A........	16.6	15.9	16.5	16.1	19.9	20.4	19.7	21.2	23.6	22.8	22.7	24.2
B........	10.7	9.1	10.4	9.3	15.0	13.1	15.5	14.3	19.1	18.5	17.9	17.7
C........	7.1	6.0	6.3	7.6	14.0	13.8	13.3	13.0	16.3	14.9	15.5	16.0
D........	7.8	5.8	6.9	6.5	13.9	13.5	12.3	12.5	18.2	16.6	16.0	15.6
E........	4.2	4.2	4.7	4.7	5.6	5.7	5.0	6.3	6.3	6.1	6.1	6.4
F........	2.6	1.8	2.3	1.9	4.5	4.7	4.6	4.8	7.6	6.9	6.7	6.3
G........	1.8	1.4	1.4	1.6	3.7	3.6	3.4	3.4	4.8	4.9	4.8	4.3
H........	0.8	0.7	0.8	1.0	2.8	2.0	2.5	3.1	4.5	4.7	3.7	3.9
I........	0.5	0.4	0.4	0.4	1.2	0.8	1.0	0.9	2.3	1.9	1.8	1.8
J........	1.7	1.6	1.7	1.6	3.1	3.0	2.9	3.4	3.7	3.7	3.7	3.6
K........	0.2	0.1	.....	.....	3.9	3.8	3.4	3.6	6.7	6.1	5.8	5.9
L........	.....	.....	.....	.....	1.7	1.5	1.2	1.4	2.3	2.4	2.0	1.7
M........	0.9	0.8	0.9	1.1	2.6	2.3	2.2	2.5	3.2	3.0	2.8	2.5
N........	.....	.....	.....	.....	0.5	0.4	0.3	0.3	1.1	1.0	0.8	0.8
O........	.....	.....	.....	.....	.....	.....	.....	.....	0.2	0.1	0.2	0.2

Set	Sixth Grade I	Sixth Grade II	Sixth Grade III	Sixth Grade IV	Seventh Grade I	Seventh Grade II	Seventh Grade III	Seventh Grade IV	Eighth Grade I	Eighth Grade II	Eighth Grade III	Eighth Grade IV
A........	25.0	25.2	25.2	25.3	26.2	28.3	29.3	28.7	30.1	27.3	30.1	27.7
B........	21.0	20.2	19.3	20.0	22.4	21.1	23.8	22.3	27.0	25.5	26.2	24.4
C........	16.3	17.7	17.4	18.1	17.4	18.3	19.4	19.7	18.5	18.6	19.9	18.9
D........	19.6	19.5	18.8	19.2	21.6	22.3	21.8	21.4	23.3	22.5	23.1	22.5
E........	6.3	6.6	6.3	7.0	7.7	7.2	7.6	7.4	8.2	78.5	7.6	7.5
F........	7.8	7.8	6.8	7.5	9.0	8.5	8.6	8.0	10.7	9.4	9.6	8.7
G........	5.3	5.3	5.1	5.2	5.8	5.6	6.1	5.6	6.8	6.2	6.0	5.9
H........	6.6	6.5	5.8	6.5	7.8	7.9	8.5	8.2	9.5	8.7	8.4	8.6
I........	3.0	3.1	2.5	2.5	4.3	3.6	3.5	3.2	5.0	4.4	4.1	3.7
J........	4.2	4.2	4.0	4.7	5.0	4.7	4.7	4.6	6.1	5.4	5.4	5.2
K........	8.6	8.4	8.2	7.6	10.7	10.2	9.9	9.6	13.2	12.1	11.6	11.1
L........	2.4	2.6	2.2	2.4	3.1	2.8	3.1	2.3	4.0	3.3	3.1	3.3
M........	3.8	3.6	3.1	3.7	4.8	4.1	4.2	3.8	5.4	4.6	4.5	4.3
N........	1.3	1.4	1.0	1.0	1.9	1.6	1.6	1.2	2.6	2.3	2.0	1.8
O........	4.6	4.3	3.4	3.5	5.9	4.8	4.4	3.9	7.5	6.0	5.5	4.8

scores for the other two groups in the third grade may be read, as well as the scores for all four groups in each of the remaining grades.

A glance at the table is sufficient to show that there is a tendency for the average score to diminish in passing from Group I to Group II, from Group II to Group III, and from Group III to Group IV in each of the sets in each of the grades. There are exceptions, of course, and the differences encountered in passing from a younger group to an older one vary in degree with the sets and with the grades.

Since the more important facts presented in Table XXXVI are presented graphically in the diagrams which follow, we shall now pass to them. In these diagrams comparisons are made between two groups only—Group I, the under-age group, and Group IV, the over-age group—because these two groups represent the extremes. In the four sections of Diagram 25 the comparisons are made between the two groups throughout the six grades in the four sets in addition, A, E, J, and M. A few very interesting differences are to be found in comparing the two curves. In the simpler sets, A and E, the over-age group seems on the whole to be superior to the under-age group, while in the more complex sets, J and M, and especially in the latter, the superiority of the under-age group is quite marked. There also seems to be a difference in the relations of the two curves in the lower and the upper grades. In the former the differences between the attainments of the groups are less in evidence than in the latter. That is, the diagram would indicate that the differences between the under-age pupils and the over-age pupils become more noticeable as we proceed upward through the grades; and this is especially true of the records made in the more complex examples.

Passing now to Diagram 26 we come to a similar comparison of records made in subtraction. The records made by the two groups in the two sets of examples in subtraction are here graphically presented. The same conclusions may be drawn from this comparison as were drawn from the comparisons of the records of the two groups in the four sets of addition, viz., that the differences are less marked in the simpler than in the more complex set and that the superiority of the under-age group increases with the progress through the grades.

The records made in the three sets in multiplication, C, G, and L, by the two groups of pupils under comparison are shown in Dia-

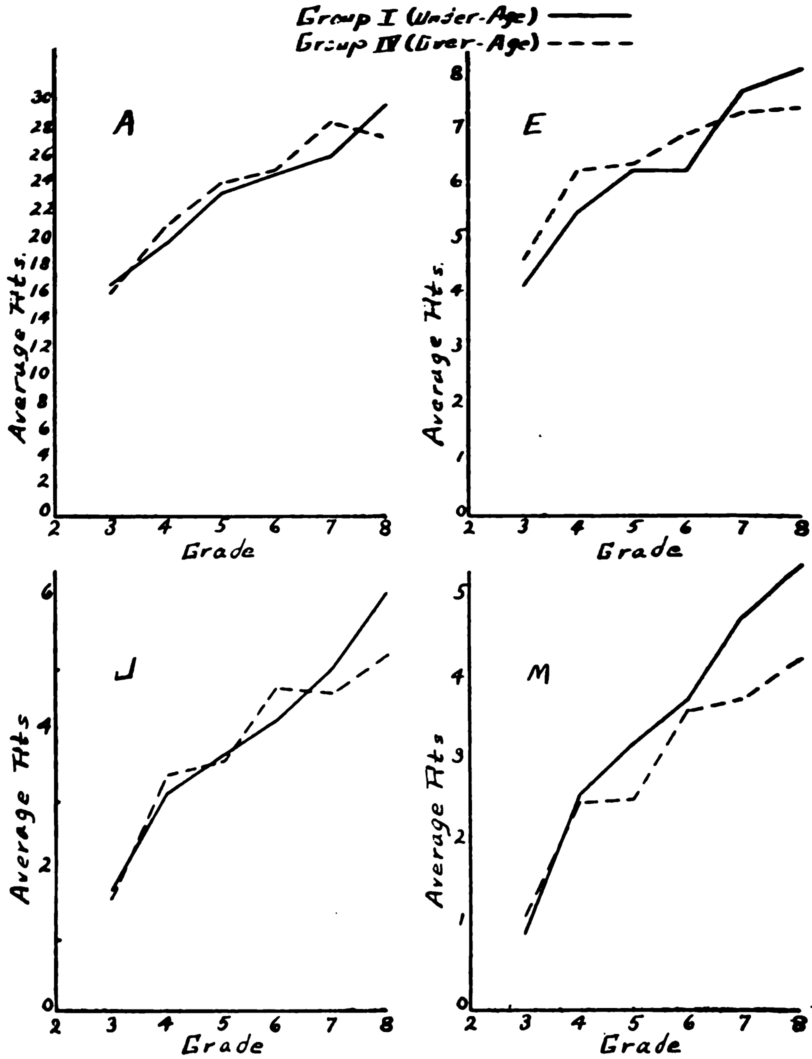


DIAGRAM 25.—A comparison of records made by two age groups through Grades 3–8 in four sets in addition (A, E, J, M).

gram 27. The differences already noted are borne out here, so that no discussion is necessary. We therefore pass to Diagram 28, in

which are graphically represented the facts for the four sets in division, D, I, K, and N. These graphs deserve especial attention

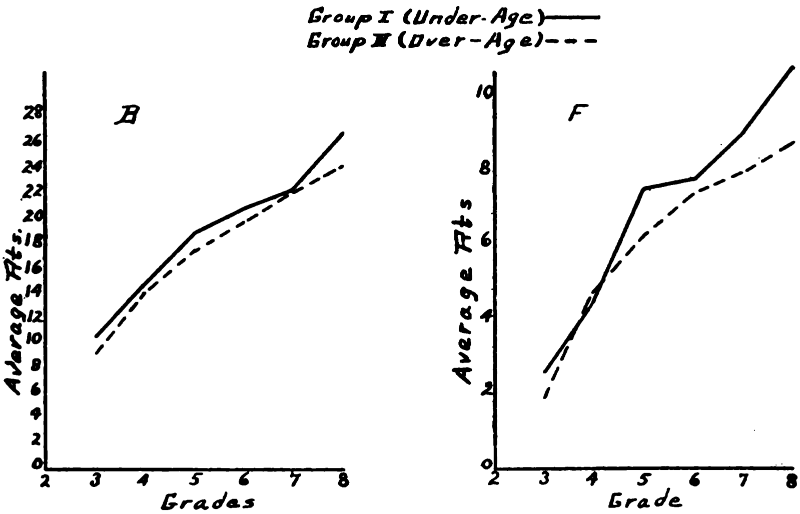


DIAGRAM 26.—A comparison of records made by two age groups through Grades 3–8 in two sets in subtraction (B, F).

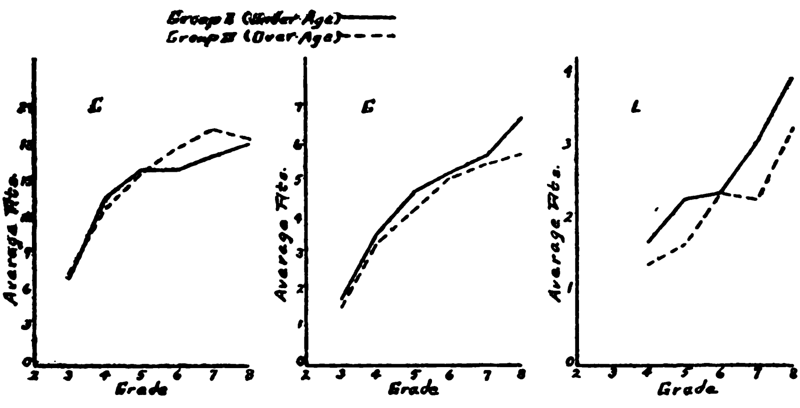


DIAGRAM 27.—A comparison of records made by two age groups through Grades 3–8 in three sets in multiplication (C, G, L).

because they so clearly represent the tendencies noted in the other sets. It should be remembered that Set D is the set of simple division combinations, Set I the set in short division, Set K the

very simple set in long division examples, and Set N the difficult set in long division. It should be borne in mind, further, that the

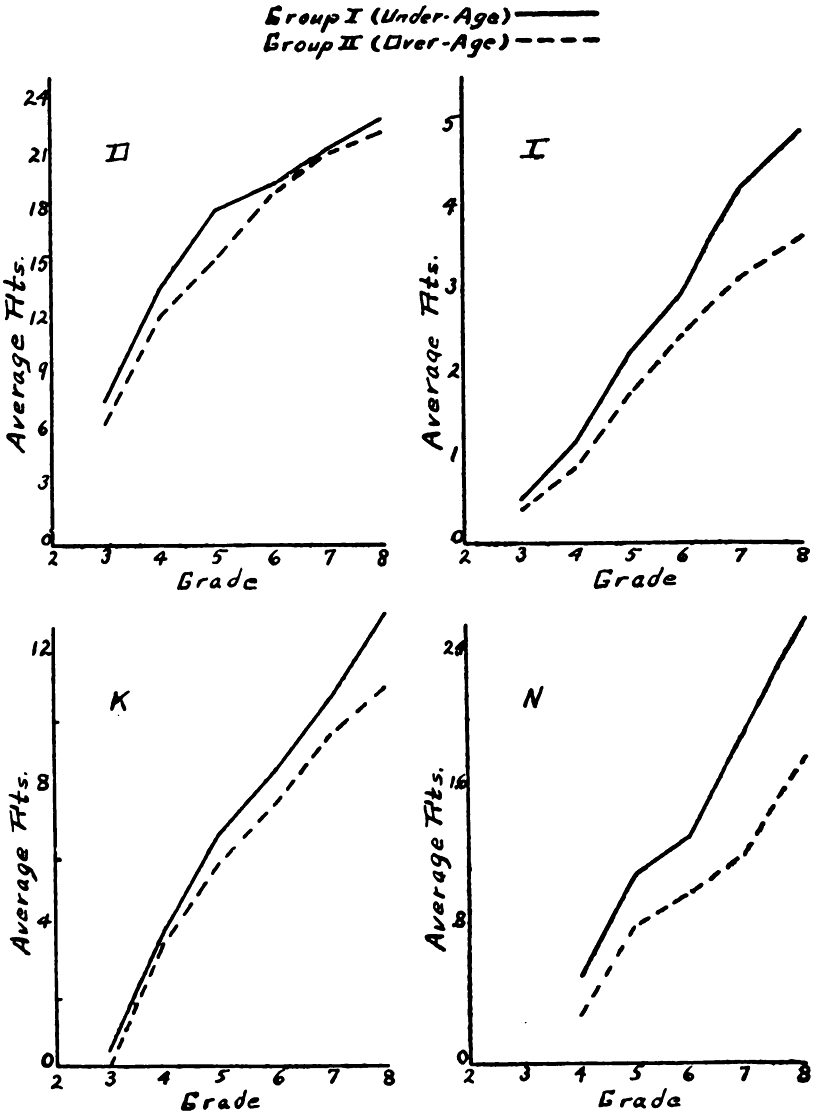


DIAGRAM 28.—A comparison of records made by two age groups through Grades 3–8 in four sets in division (D, I, K, N).

returns on accuracy in chapter iii showed Set I to be more difficult than Set K, owing to the introduction into the examples of the former set of the operation of "carrying." In order of complexity and of difficulty, therefore, the four sets should be arranged thus: D, K, I, and N. If we proceed in this order, it is very clearly shown that as we pass from a less complex to a more complex type of operation the superiority of the under-age group becomes more and more obvious. And furthermore, as before indicated, these graphs very plainly show that this superiority of the under-age group increases with progress through the grades.

Diagram 29 is of interest because it is a diagram presenting the facts for Set O, the most complex of the sets and the set worked with the greatest percentage of error. In this diagram, more than in any of the others, the superiority of the under-age group is indicated.

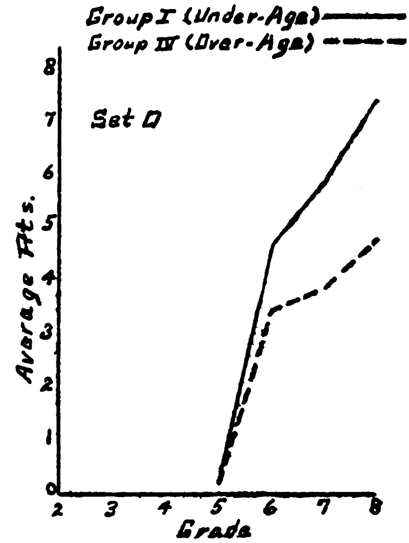


DIAGRAM 29.—A comparison of records made by two age groups through Grades 3–8 in fractions (Set O).

A diagram of a type quite different from the preceding is found in Diagram 30, in which the attempt has been made to make a cross-section of the records of Groups I and IV in the eighth grade. In order to do this, it was necessary to resort to the system of weights derived in chapter iii. Under this system the average "rights" for each of these groups in each of the sets has been converted into terms of the standard "unit." On the basis of the average number of "units" thus made in each set, a curve is drawn to represent each group. The diagram is understood if it be borne in mind that the distance of the curve above the horizontal axis is proportionate to the number of "units" made in the particular set indicated by the group which the curve represents. This diagram brings out

more clearly than any of the others thus far examined the increasing superiority of the under-age group as we proceed from the less to the more complex types of operation. This is indicated, with exceptions of course, by the increasing divergence of the two curves as we pass from left to right in the diagram.

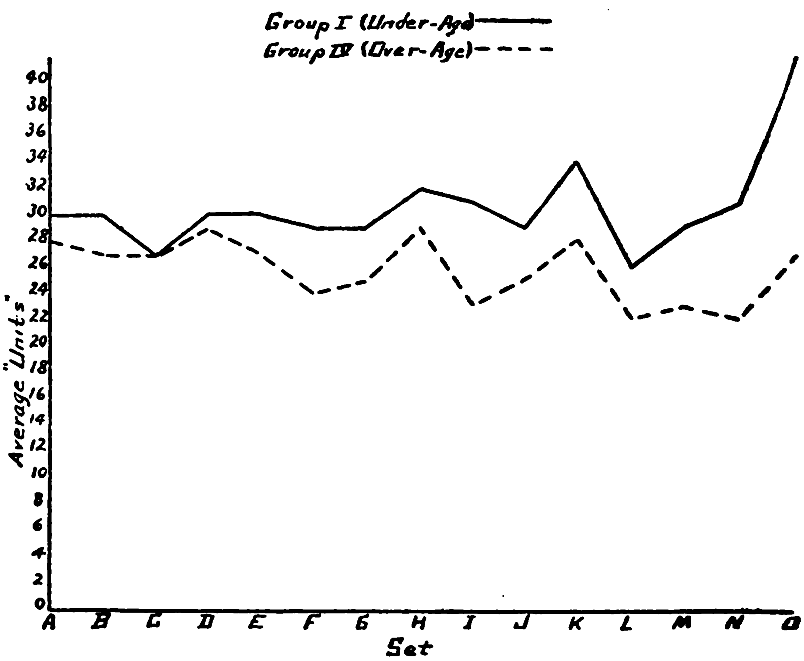


DIAGRAM 30.—Average "units" made in each set by two age groups in eighth grade.

This same matter is approached from a slightly different angle in Table XXXVII and in Diagram 31. By use of the system of weights just referred to, the average "rights" made by each of the groups in each of the sets throughout the six grades has been converted into the terms of the "unit." The 15 sets were then classified into six groups on the basis of complexity. Into the first group were put Sets A, B, C, and D, for the examples of these sets are clearly the most simple examples in the test. Into the second group were put Sets E, F, and G, representing the examples of the

next degree of complexity. Set I was not included in this group because the returns seemed to indicate that it is of much greater difficulty than any of the other three sets. Set H was put in a class by itself because of the peculiar type of reactions made to it by the pupils. Sets I, J, and K were then put into the fourth

TABLE XXXVII

AVERAGE NUMBER OF "UNITS" MADE IN CERTAIN GROUPS OF SETS BY EACH OF FOUR AGE GROUPS IN GRADES 3–8. DATA FROM 2,400 PUPILS

Set	Third Grade				Fourth Grade				Fifth Grade			
	I	II	III	IV	I	II	III	IV	I	II	III	IV
A, B, C, D	49	42	47	45	75	72	70	72	92	87	85	87
E, F, G	30	26	29	29	48	50	46	51	65	62	61	59
H	3	2	3	3	9	7	8	10	15	16	12	13
I, J, K	11	10	10	10	32	29	29	31	49	45	44	43
L, M, N	5	4	5	6	31	27	24	26	45	44	38	34
O	....	....	....	....	....	....	....	....	11	6	11	11
Total	98	84	94	93	195	185	177	190	277	260	251	247

Set	Sixth Grade				Seventh Grade				Eighth Grade			
	I	II	III	IV	I	II	III	IV	I	II	III	IV
A, B, C, D	97	99	96	97	103	107	112	110	117	112	117	112
E, F, G	67	68	64	68	78	73	76	73	88	80	79	76
H	22	22	19	22	26	27	29	28	32	29	28	29
I, J, K	60	60	55	56	77	70	69	66	94	84	80	76
L, M, N	52	53	43	48	69	59	61	49	86	75	68	67
O	26	24	19	19	33	27	24	22	42	33	31	27
Total	324	326	296	310	386	363	371	348	459	413	403	386

group because they were considered to be less complex than the last three sets in the fundamentals, L, M, and N, which were put into the fifth group. Set O was, like Set H, kept by itself because it is different from the other sets, the examples being more complex and having been worked with a larger percentage of error than those of the other sets. Although there may be serious question concerning some of these groups, the writer is of the opinion that the four groups composed of Sets A, B, C, and D, Sets E, F, and G, Sets L, M, and N, and Set O, respectively, do represent groups of examples of increasing complexity.

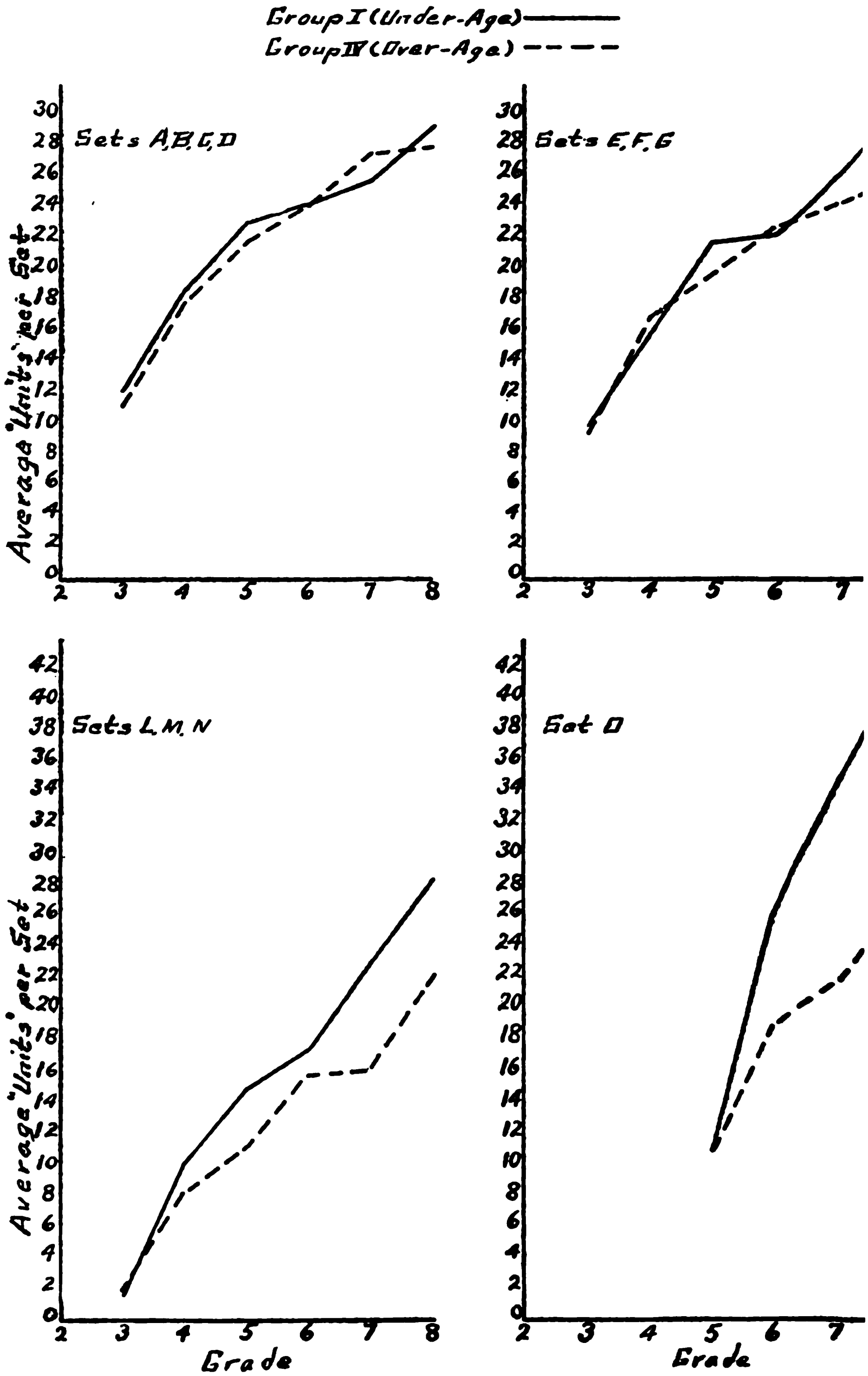


DIAGRAM 31.—A comparison of average numbers of "units" made in certain groups of sets by two age groups through Grades 3–8.

Thinking this last statement to be a safe proposition, Diagram 31 has been constructed, in which the average records made by the under-age and the over-age groups in these four groups of sets are compared. Thus, as we pass from one section of the diagram to the next, we proceed from records made in sets of simple examples to records made in more complex examples. Here again the fact is very clearly brought out that the superiority of the under-age group is not at all conspicuous in the simpler operations, but becomes more and more marked as the more complex operations are encountered.

One more diagram should be presented before we leave this phase of the problem for the purpose of bringing out more clearly the differences met with as progress is made through the grades. In order to bring the cumulative force of records made in the entire test to bear on this problem, the average numbers of "units" made by each of the age-groups in the 15 sets are added to get a single score to represent each group in each grade. A graphic representation of this comparison is shown in Diagram 32, from which it is plain that the difference between the total scores made by the under-age and the over-age groups becomes consistently greater (except at the sixth grade) from grade to grade.

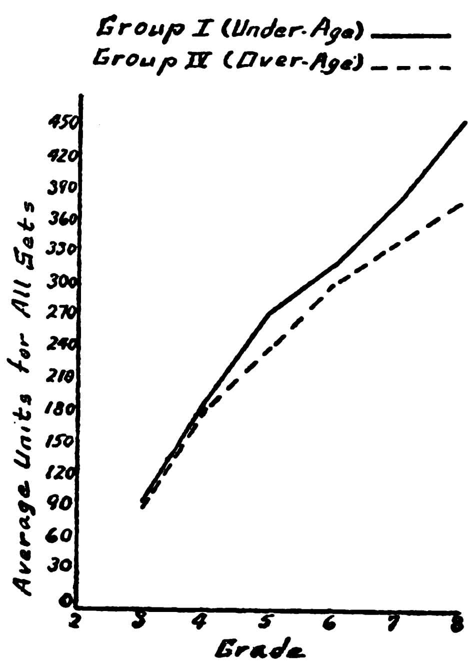


DIAGRAM 32.—A comparison of the average numbers of "units" made in all sets through Grades 3–8 by two age groups.

Before closing the comparison between the age groups on the basis of the number of examples worked correctly, two summary

tables should be presented, Tables XXXVIII and XXXIX. In the first is presented the average number of "rights" made in each set by the four age groups in Grades 3–8 combined. By applying our own system of weights to this table we obtain the second, in

TABLE XXXVIII

AVERAGE "RIGHTS" IN EACH SET FOR EACH OF FOUR AGE GROUPS IN GRADES 3–8 COMBINED

Set	Average Rights			
	I	II	III	IV
A	23.6	23.3	23.9	23.9
B	19.2	17.9	18.5	18.0
C	14.9	14.9	15.3	15.5
D	17.4	16.7	16.5	16.3
E	6.4	6.2	6.2	6.5
F	7.0	6.5	6.4	6.2
G	4.7	4.5	4.4	4.3
H	5.3	5.1	5.0	5.2
I	2.7	2.3	2.2	2.1
J	4.0	3.8	3.7	3.8
K	7.2	6.8	6.5	6.3
L	2.3	2.1	1.9	1.8
M	3.5	3.1	3.0	3.0
N	1.2	1.1	1.0	0.9
O	3.0	2.5	2.3	2.1

which is found a statement of the average number of "units" made in the entire test of 15 sets by each of these age groups in the combined six grades. These tables bring out nothing that has not

TABLE XXXIX

AVERAGE "UNITS" MADE IN EACH OF FOUR AGE GROUPS IN ALL SETS BY GRADES 3–8

	Group			
	I	II	III	IV
Average "units" made.	290	272	265	262

already been discussed, but merely present in summary form what has already been included in the other tables. They may therefore be passed by without further comment.

Now, to sum up briefly the facts that have been discovered with reference to the number of examples worked correctly by the pupils

TABLE XL

AVERAGE "ATTEMPTS" IN EACH SET FOR EACH OF FOUR AGE GROUPS IN GRADES 3–8. DATA FROM 2,400 PUPILS

Set	Third Grade				Fourth Grade				Fifth Grade			
	I	II	III	IV	I	II	III	IV	I	II	III	IV
A	17.3	16.6	17.1	17.1	20.1	20.6	19.9	21.7	24.0	22.9	22.9	24.5
B	11.5	9.9	11.1	10.6	15.4	13.4	14.1	15.0	19.3	19.0	18.3	18.2
C	8.6	7.2	7.5	9.0	15.0	15.3	14.5	14.7	17.6	16.7	17.2	18.0
D	9.0	7.1	8.2	7.9	14.5	14.5	13.1	13.7	18.6	17.2	17.0	16.7
E	4.9	4.7	5.3	5.4	6.1	6.2	5.5	7.1	6.8	6.6	6.7	6.9
F	4.0	3.5	3.7	3.8	5.7	6.0	6.0	6.6	8.3	7.7	7.6	7.7
G	2.6	2.4	2.3	2.8	4.3	4.3	4.2	4.4	5.5	5.6	5.6	5.4
H	1.8	1.5	1.8	1.8	3.7	3.0	3.4	4.0	6.3	6.5	6.0	5.8
I	1.4	1.5	1.3	1.6	2.2	1.9	2.1	2.2	3.1	2.7	2.8	2.7
J	2.9	2.8	3.2	3.2	4.3	4.5	4.4	5.0	5.0	4.8	5.1	5.0
K	....	....	....	....	4.8	4.6	4.4	4.9	7.2	6.8	6.7	7.1
L	....	....	....	....	3.0	2.9	2.7	3.3	3.7	3.6	3.6	3.4
M	2.1	1.9	2.0	2.3	3.7	3.5	3.6	4.2	4.5	4.2	4.3	4.2
N	....	....	....	....	1.3	1.2	1.3	1.6	1.9	1.6	1.7	1.7
O	....	....	....	....	0.5	0.5	0.3	0.2	1.0	0.8	1.3	1.7

Set	Sixth Grade				Seventh Grade				Eighth Grade			
	I	II	III	IV	I	II	III	IV	I	II	III	IV
A	25.2	25.4	25.7	25.7	26.4	28.6	29.7	29.0	30.4	27.7	30.4	28.1
B	21.3	20.5	19.7	20.5	22.7	21.4	24.3	22.9	27.2	25.8	26.5	24.7
C	18.4	20.0	19.6	20.7	19.3	21.0	22.5	22.8	21.0	21.2	22.3	21.4
D	20.0	20.0	19.4	20.0	21.9	22.9	22.5	22.2	23.8	23.1	23.9	23.1
E	6.7	7.0	6.8	7.5	8.2	7.8	8.1	8.1	8.5	8.0	8.2	8.0
F	8.5	8.6	8.0	8.7	9.9	9.8	9.9	9.5	11.6	10.5	10.6	9.9
G	5.9	6.2	5.8	6.2	6.6	6.5	6.9	6.9	7.5	7.0	6.9	6.8
H	9.2	9.3	8.9	9.8	10.1	10.9	11.7	11.4	11.9	10.8	11.8	11.1
I	3.8	3.8	3.4	3.6	4.9	4.3	4.4	4.4	5.6	5.1	5.0	4.5
J	5.5	5.6	5.3	6.1	6.5	6.1	6.3	6.3	7.4	7.0	7.1	6.8
K	9.3	9.0	9.0	9.1	11.4	11.0	10.7	10.8	13.5	12.6	12.4	11.8
L	3.6	3.9	3.7	4.1	4.4	4.6	4.6	4.4	5.3	5.0	4.7	4.8
M	4.9	5.2	4.6	5.4	6.1	5.5	5.8	5.7	6.8	6.1	5.9	6.0
N	1.9	2.2	2.0	2.0	2.4	2.4	2.5	2.3	3.0	2.8	2.6	2.5
O	7.2	7.5	7.3	7.8	8.4	7.9	8.4	8.0	9.3	8.9	8.7	8.4

in these four age groups, it may be said: (1) that there are differences, (2) that on the average the younger groups are superior to

the older, (3) that this superiority is more marked in the later than in the earlier grades, and (4) that it is also more marked in the handling of the more complex than in the handling of the simpler types of examples.

We pass now to an examination of the records made by the pupils in these four age groups for the purpose of discovering differences in the number of examples attempted in the various sets. These facts are presented in detail in Table XL. Since this table is identical in form with tables already explained, our attention may be directed at once to the facts themselves. A glance is sufficient to show that no such differences are to be found in the comparison of the "attempts" made by the four groups as were found in the comparison of the "rights." Especially is it true that in the simpler sets there is no tendency for the under-age pupils to attempt more examples than do the over-age pupils. In the more complex sets, however, there is such a tendency, especially in the upper grades, but to a much less marked degree than in the case of the "rights."

Let us therefore turn to Table XLI and Diagram 33, in which are presented the total scores attempted in the entire test by each of the groups in each of the grades. This total score has been

TABLE XLI

AVERAGE "UNITS" ATTEMPTED IN ALL SETS BY EACH OF FOUR AGE GROUPS IN GRADES 3–8

Grade	Group			
	I	II	III	IV
3........	133	121	130	136
4........	250	243	239	267
5........	327	307	311	312
6........	390	403	386	412
7........	454	447	466	454
8........	517	486	489	469

determined by using the system of weights already employed in obtaining total scores of "rights." The diagram shows very clearly that there is no clear difference between the two extreme groups. In three of the six grades the over-age group attempts more

"units" than does the under-age group, while in only two grades is the reverse the case.

In Tables XLII and XLIII the facts concerning attempts are put in summary form. In the first table we have the average number of examples attempted in each set by each of the four age groups in Grades 3–8 combined, while in the second table these set averages are reduced to a single score by the use of the system of weights. Table XLII shows no clear tendency of any one group to take the lead; one group forges ahead in one set only to fall back in another. Table XLIII likewise shows no differences worth considering.

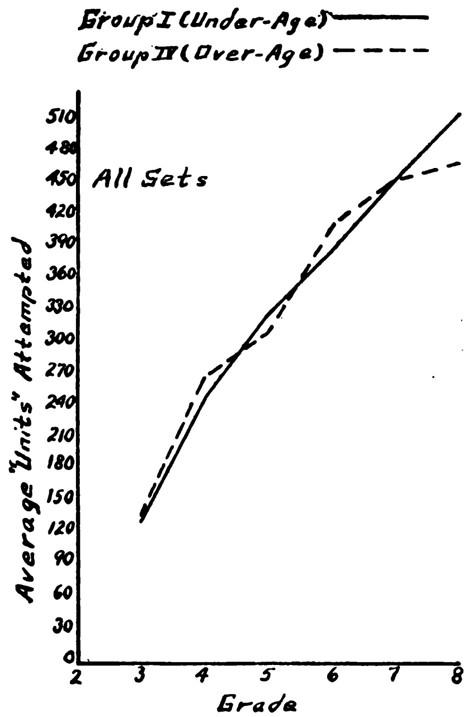


DIAGRAM 33.—A comparison of the average numbers of "units" attempted in all sets through Grades 3–8 by two age groups.

In summary it may therefore be said that the study reveals no clear differences between any two of the four age groups in numbers of examples attempted in the various sets.

A third phase of the problem now presents itself, although it is implied in what has gone before, and that is the question of accuracy. Of course, since it has been found that the under-age pupils excel in "rights" and on the average attempt no more than do the over-age pupils, it necessarily follows that the former have attained a greater degree of accuracy. However, since this is only a general impression, it will be worth while to make a special study of accuracy.

In Table XLIV there is presented the percentage of accuracy made in each set by each of the four age groups in Grades 3–8. An examination of this table shows the differences to be of about the

TABLE XLII

AVERAGE "ATTEMPTS" IN EACH SET FOR EACH OF FOUR AGE GROUPS IN GRADES 3–8 COMBINED

Set	Average Attempts			
	I	II	III	IV
A	23.9	23.6	24.3	24.4
B	19.6	18.3	19.0	18.6
C	16.6	16.8	17.3	17.8
D	18.0	17.5	17.4	17.3
E	6.8	6.7	6.8	7.2
F	8.0	7.7	7.6	7.7
G	5.4	5.3	5.3	5.4
H	7.2	7.0	7.3	7.3
I	3.5	3.2	3.2	3.2
J	5.3	5.1	5.2	5.4
K	7.7	7.3	7.2	7.3
L	3.4	3.3	3.2	3.4
M	4.7	4.4	4.4	4.6
N	1.7	1.7	1.7	1.7
O	4.4	4.3	4.3	4.4

same order as those found in the comparison of the groups in the number of examples worked correctly. In the simpler sets there is not a great deal of difference between the under-age and the

TABLE XLIII

AVERAGE UNITS ATTEMPTED IN EACH OF FOUR AGE GROUPS IN ALL SETS BY GRADES 3–8

	Group			
	I	II	III	IV
Average "units" attempted	345	335	337	342

over-age pupils, while in the more complex examples the difference becomes accentuated. The differences also appear to be greater in the upper than in the lower grades, although in this respect there

is less difference between the earlier and the later grades than was found to be true in the case of the "rights."

TABLE XLIV

PERCENTAGE OF ACCURACY IN EACH SET FOR EACH OF FOUR AGE GROUPS IN GRADES 3–8. DATA FROM 2,400 PUPILS

Set	Third Grade				Fourth Grade				Fifth Grade			
	I	II	III	IV	I	II	III	IV	I	II	III	IV
A........	96.2	96.1	96.1	94.2	98.9	98.8	98.7	97.9	98.5	99.4	98.8	98.8
B........	93.0	92.2	94.0	88.4	97.3	97.6	95.2	95.5	99.1	97.5	97.9	97.1
C........	82.0	83.5	84.0	84.7	93.2	90.3	91.4	88.1	93.0	89.4	90.5	88.7
D........	85.8	82.3	84.4	82.2	95.8	93.0	93.9	91.2	97.5	96.6	93.8	93.2
E........	86.2	89.5	87.8	85.7	92.4	91.1	90.3	89.9	92.7	91.7	91.0	92.9
F........	64.5	50.9	62.2	49.3	79.3	77.0	76.1	72.6	90.8	89.0	87.9	82.2
G........	67.6	60.9	60.8	57.3	85.0	83.9	79.8	77.7	86.7	88.8	85.8	80.1
H........	46.0	45.4	43.5	52.2	74.8	68.2	73.3	78.8	71.4	73.1	62.6	65.9
I........	31.2	25.2	28.0	27.7	54.8	41.5	48.5	41.1	74.4	68.5	63.3	65.0
J........	58.4	56.5	54.5	50.2	72.1	65.9	64.4	67.1	73.5	76.2	74.0	71.4
K........	28.6	55.6	....	....	82.0	84.0	76.1	73.3	93.3	89.9	86.2	83.0
L........	....	....	....	....	56.0	52.4	43.8	42.0	63.5	64.8	57.2	50.7
M........	44.1	43.4	46.7	47.4	70.4	63.9	59.6	61.1	71.6	71.4	65.9	58.4
N........	....	....	....	....	40.9	31.1	25.2	21.0	60.5	60.4	47.9	46.5
O........	....	....	....	....	....	....	....	....	16.5	14.8	11.9	8.7

Set	Sixth Grade				Seventh Grade				Eighth Grade			
	I	II	III	IV	I	II	III	IV	I	II	III	IV
A........	99.3	99.3	98.0	98.3	99.2	99.1	98.5	98.8	99.0	98.4	99.0	98.7
B........	98.7	98.6	97.8	97.6	98.7	98.6	97.9	97.4	99.3	98.8	98.8	98.8
C........	88.6	89.8	88.5	87.1	90.2	87.0	86.3	86.4	88.1	87.8	89.1	88.6
D........	98.1	97.3	96.8	95.7	98.5	97.3	96.8	96.2	97.8	97.4	96.6	97.3
E........	94.4	93.7	92.8	92.9	95.0	93.0	93.5	90.8	95.9	94.2	93.0	93.5
F........	90.9	89.8	85.3	86.6	90.2	87.5	87.1	84.2	92.4	89.9	90.5	87.9
G........	89.4	86.7	86.3	84.5	88.2	85.3	88.2	81.3	89.8	89.4	87.3	86.8
H........	71.9	70.2	65.7	66.5	76.8	72.4	72.8	71.9	79.7	80.2	71.0	77.0
I........	80.6	80.1	74.0	68.1	86.4	83.4	78.9	71.9	87.8	85.7	81.8	80.4
J........	77.4	75.7	76.0	77.9	77.9	77.0	75.1	72.9	82.2	77.2	76.0	76.3
K........	92.4	93.4	91.5	83.6	94.0	92.6	92.6	88.8	97.3	95.9	93.5	94.0
L........	66.1	66.4	60.4	58.5	69.8	61.5	66.7	51.8	76.1	65.9	64.7	68.3
M........	76.2	68.5	67.5	67.7	79.5	73.1	71.2	66.3	80.1	74.5	76.4	71.3
N........	69.0	63.9	51.3	50.5	77.3	66.4	63.4	52.4	86.9	84.1	77.3	74.5
O........	63.9	56.4	46.6	44.3	69.6	60.7	53.0	48.2	81.1	67.2	63.7	57.4

In Diagram 34 the under-age and the over-age groups are compared through the six grades in Sets L, M, N, and O. These graphs

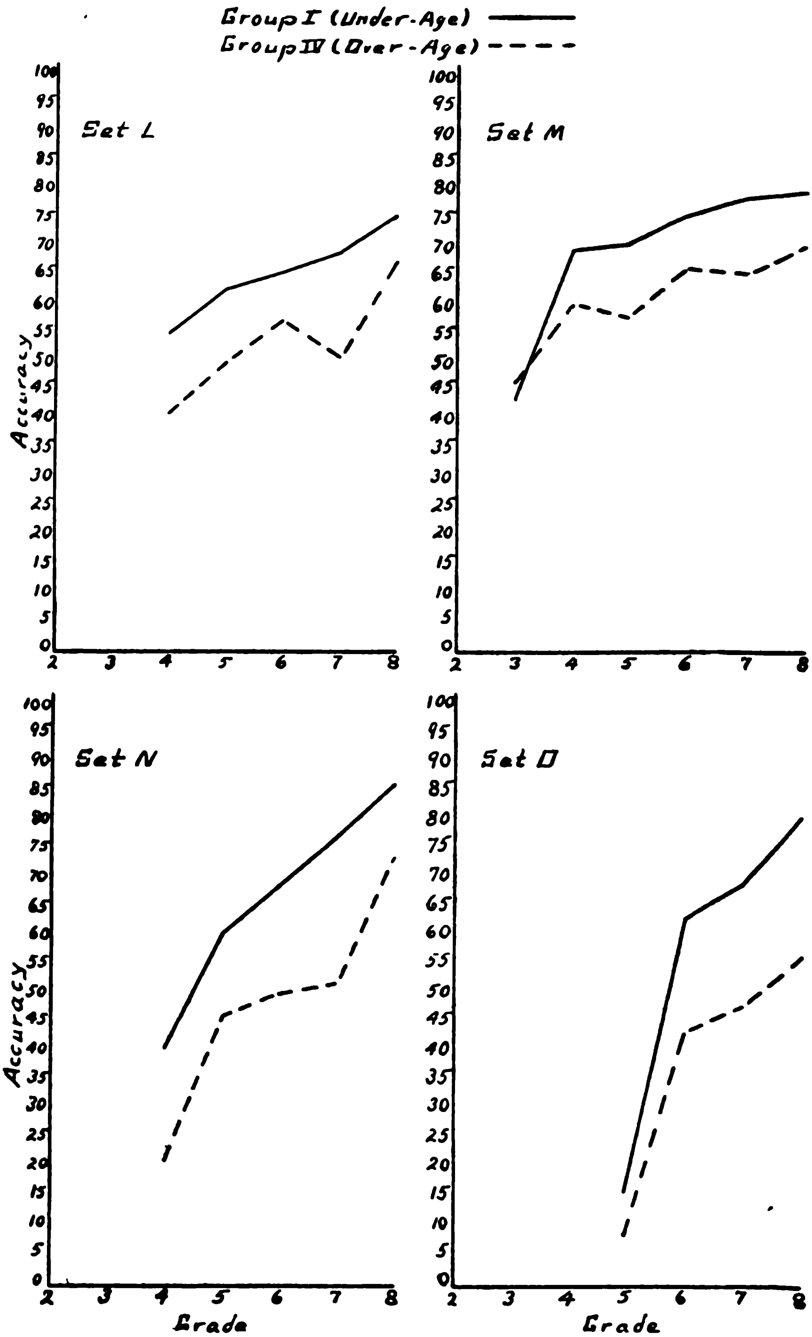


DIAGRAM 34.—A comparison of percentages of accuracy made in Sets L (multiplication), M (addition), N (division), and O (fractions) by two age groups through Grades 3–8.

show very distinct differences between the two groups, and these differences seem to be most marked in Sets N and O.

On the basis of the average number of "units" made in the entire test by the four groups in the several grades, presented in Table XXXVII, and the average number of "units" attempted, presented in Table XLI, it was possible to determine the percentage of accuracy made by each of the age groups in the entire test. These measures of accuracy are presented in Table XLV and in

TABLE XLV

AVERAGE ACCURACY IN ALL SETS BY EACH OF FOUR AGE GROUPS IN GRADES 3–8

Grade	Group			
	I	II	III	IV
3........	73.7	69.4	72.3	68.4
4........	78.0	76.1	74.1	71.2
5........	84.7	84.7	80.7	79.2
6........	83.1	80.9	76.7	75.2
7........	85.0	81.2	79.6	76.7
8........	88.8	85.0	82.4	82.3

TABLE XLVI

AVERAGE ACCURACY FOR EACH SET FOR EACH OF FOUR AGE GROUPS IN GRADES 3–8 COMBINED

Set	Average Accuracy			
	I	II	III	IV
A........	98.6	98.5	98.3	98.1
B........	98.1	97.7	97.4	96.6
C........	89.6	88.4	88.6	87.4
D........	96.6	95.6	94.9	94.2
E........	93.3	92.4	91.6	91.2
F........	87.5	84.7	84.2	80.6
G........	86.4	85.0	84.3	80.3
H........	74.3	72.8	68.3	71.1
I........	76.8	72.6	69.6	64.9
J........	75.5	73.2	71.7	71.2
K........	93.3	92.3	89.9	86.3
L........	67.3	62.4	59.6	54.8
M........	74.0	68.9	67.6	64.1
N........	71.1	65.5	57.2	51.6
O........	69.3	59.2	52.2	47.3

Diagram 35. In a previous paragraph the comment was made that differences in accuracy between the two extreme age groups did not increase in the same degree with progress through the grades as was found to be true in the case of differences in "rights." This point is brought out very clearly in Diagram 35, which shows that, while there is some slight increase in the superiority in accuracy of the under-age group over the over-age group, as we pass up through the grades the increase is of small significance.

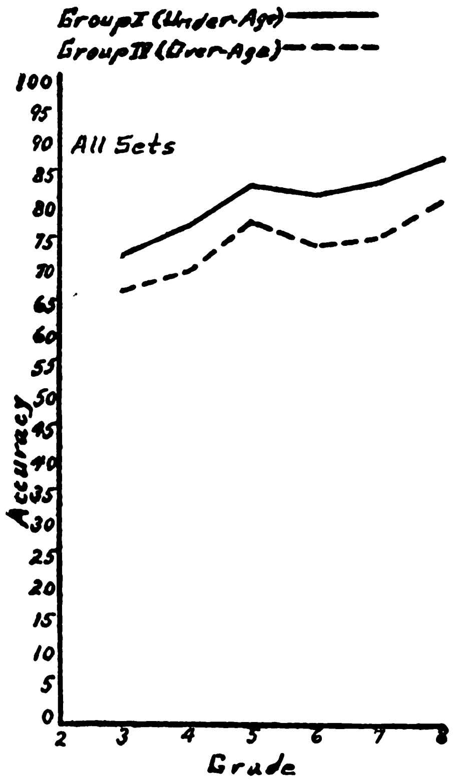


DIAGRAM 35.—A comparison of percentages of accuracy made in all sets by two age groups through Grades 3-8.

Table XLVI is a summary in which there is presented the average accuracy made in each set by the four groups in the six grades combined. An examination of this table shows it to be quite remarkable, for there are only two cases in the entire table where passing from the percentage of accuracy made in a particular set by one of the age groups to the percentage of accuracy made by the next older group is not accompanied by a decrease in accuracy. The two exceptions are: (1) in Set C the accuracy of Group II is 88.4 per cent and that of Group III is 88.6 per cent; (2) in Set H the accuracy of Group III is 68.3 per cent and that of Group IV is 71.1 per cent. From this it is certainly evident that on the average the younger pupils of a grade are more accurate than the older pupils.

The facts presented in Table XLVI for Groups I and IV are presented in Diagram 36 in graphic form. The one thing emphasized by this diagram, in addition to the mere fact that the under-age group is more accurate than the over-age group, is that the difference becomes more marked as we pass from the simpler to the more complex examples, until the greatest difference is found in Set O, fractions.

DIAGRAM 36.—Average accuracy made in each set by each of two age groups in Grades 3–8 combined.

Table XLVII presents a final statement of the relative accuracy of the four groups in all the sets in Grades 3–8 combined. These facts merely emphasize what has already been said, and they need be discussed no further.

As a brief summary of the facts relating to the accuracy of the four age groups in the tests it may be said: (1) that there are differences; (2) that these differences show the younger pupils to be on the average more accurate in their work than the older pupils;

(3) that these differences are on the whole quite uniform from grade to grade; and (4) that the differences are more evident in the more complex than in the simpler examples.

TABLE XLVII

AVERAGE ACCURACY IN EACH OF FOUR AGE GROUPS IN ALL SETS IN GRADES 3-8

	Group			
	I	II	III	IV
Accuracy.........	84.0	81.3	78.8	76.8

PROMOTION GROUPS

The purpose of this second division of the investigation is to throw some light on promotion practices, as found in Grand Rapids, and their relations to arithmetical attainments. Although the paucity of the data has made it quite impossible to make the case conclusive in any instance, the attempt is made in this study to do four things: (1) to determine differences in arithmetical attainments of three promotion groups in the eighth grade—the fast, the regular, and the slow group; (2) to determine differences among these same pupils when regrouped on the basis of age, for the purpose of finding out whether or not the age or the promotion factor is the more important; (3) to determine differences in arithmetical attainments of three other promotion groups in the seventh grade, "regular" pupils (those making normal progress), "irregular" pupils (those repeating because of transfer of schools, sickness, etc.), and failures (those repeating because of failure to do the work of the grade); and (4) to determine differences in arithmetical attainments of two more promotion groups in the eighth grade, the one composed of pupils failing below the sixth grade, the other of those failing above the fifth grade.

METHOD

This study is based entirely on records made by children in the Grand Rapids schools. An examination of the arithmetic folder used in the survey of that city shows that on the front page of the

folder the following question was asked of the pupil taking the test: "Have you ever repeated the arithmetic of a grade because of non-promotion or transfer from other school? If so, name grade. . . . . Explain cause." In addition to answers to this question, through special request the children from a number of the schools indicated the fact whether they had ever skipped one or more grades. Thus records were secured showing normal progress, progress below normal, and progress above normal, and the cause of slow progress—repeating—was ascertained in the cases where it occurred.

In comparing the fast, regular, and slow pupils the method of selecting records of pupils employed in connection with the study of the age groups was adopted for the purpose of eliminating the factors of differences in giving the test, and differences in training. The same method was used in selecting records for the other promotion groups. In this way 150 records were secured for the first division of the study, representing 50 "fast" pupils, 50 "regular" pupils, and 50 "slow" pupils. For the second division of the study these same 150 records were used, merely being grouped in a different way. For the third division of the study, 162 records were secured, 54 for each group. The data for the last part of the study were the most meager, for it was possible to obtain only 32 records for each of the groups.

The records thus selected were carefully scored; the attempts and rights were tabulated; and average attempts, average rights, and percentages of accuracy were computed.

RESULTS

As already stated, the first division of this study relates to three promotion groups, the first designated as the "fast" group, composed of pupils who have skipped one or more grades; the second designated as the "regular" group, made up of pupils who have neither skipped nor repeated; and the third designated as the "slow" group, composed of pupils who have repeated one or more grades. Each group is represented by 50 pupils in Grade 8-2.

The average number of examples worked correctly in each set by the pupils in each of these groups is shown in Table XLVIII.

The total number of "units" made by each of the groups, as determined by the system of weights, is also presented in this table. In general, the differences found here are of the same order as those discovered in the study of the age groups. The "fast" pupils are decidedly superior to the "slow" pupils, while the "regular" pupils occupy an intermediate position. These differences are more marked in the more complex than in the simpler examples.

TABLE XLVIII

AVERAGE "RIGHTS" MADE IN EACH SET BY THREE PROMOTION GROUPS. GRADE 8-2. DATA FROM 150 PUPILS

Group	Set															Total Units
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	
Fast	30.6	25.1	21.1	22.7	7.7	10.9	6.8	8.4	4.1	5.9	10.7	4.9	5.8	2.3	6.6	442
Regular	29.9	24.1	20.7	22.8	7.8	10.1	6.7	8.5	4.1	5.3	10.7	4.4	5.1	1.8	5.3	414
Slow	30.2	26.1	20.6	22.4	7.2	10.3	5.9	8.1	3.7	5.4	10.2	4.1	5.2	1.7	4.7	399

The facts for the "attempts," made by the same groups, appear in Table XLIX. Although the differences between the groups are not quite so marked here as in the case of the "rights," they are very substantial, being larger than those found in the study of the Cleveland age group.

TABLE XLIX

AVERAGE "ATTEMPTS" MADE IN EACH SET BY THREE PROMOTION GROUPS. GRADE 8-2. DATA FROM 150 PUPILS

Group	Set															Total Units
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	
Fast	30.7	25.4	22.2	23.4	8.2	11.7	7.6	11.4	4.9	7.5	10.9	6.6	7.2	2.9	9.1	516
Regular	30.2	24.4	21.7	23.3	7.6	10.8	7.5	10.9	4.9	7.0	11.0	6.2	6.7	2.9	8.2	492
Slow	30.5	26.5	22.1	23.1	7.7	11.4	6.9	11.4	4.9	6.9	10.4	5.7	6.7	2.7	7.9	487

The average accuracy achieved by the three groups in all the sets is shown in Table L. As would be suspected from the facts presented concerning "rights" and "attempts," the "fast" group is the most accurate.

In order to discover the relative importance of the factors of promotion and of age, the 150 pupils used in the study just discussed were regrouped on the basis of age, the 50 youngest being placed in one group, the 50 oldest in a second group, and the remaining 50 in a third group.

TABLE L

AVERAGE ACCURACY IN ALL SETS. THREE PROMOTION GROUPS. GRADE 8-2

	Accuracy
Fast	85.7
Regular	84.2
Slow	81.9

Tables LI, LII, and LIII are identical in form with the three tables just discussed in connection with the age groups, the first presenting the facts for the "rights," the second those for the "attempts," and the third those for the accuracy of each of these three age groups. The very interesting and surprising fact brought

TABLE LI

AVERAGE "RIGHTS" MADE IN EACH SET BY THREE AGE GROUPS. GRADE 8-2. DATA FROM 150 PUPILS

Group	Set															Total Units
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	
Young	31.1	26.8	20.9	24.2	8.0	11.1	7.1	9.1	4.3	5.6	11.2	4.9	5.6	2.3	6.8	452
Normal	30.9	23.6	21.6	22.6	7.7	10.3	6.8	8.8	4.3	6.1	10.6	4.7	5.7	2.0	5.8	431
Old	28.7	22.7	20.0	21.2	6.9	9.9	5.6	6.9	3.3	4.9	9.7	3.6	4.8	1.6	4.2	369

out by these tables on the promotion groups is that the differences between the "young" and the "old" groups are greater than the differences between the "fast" and the "slow" groups. That is, the younger pupils of the grade are more superior to the older pupils of the grade than the "fast" pupils are to the "slow" pupils. Of course the data do not warrant any definite conclusions, but the indications are that, so far as attainments in arithmetic are concerned, pupils may fail of promotion, or rather may not be

recommended for extra promotion, because they are thought to be too young rather than because of their inability to do more advanced work.

TABLE LII

AVERAGE "ATTEMPTS" MADE IN EACH SET BY THREE AGE GROUPS. GRADE 8-2. DATA FROM 150 PUPILS

Group	Set															Total Units
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	
Young	31.3	27.0	21.7	24.7	8.3	11.8	7.8	11.6	5.1	7.7	11.4	6.8	7.1	3.1	9.0	524
Normal	31.3	24.0	22.8	23.3	8.2	11.1	7.6	11.7	5.2	7.4	11.0	6.3	7.2	2.9	8.7	511
Old	28.9	23.1	21.5	21.7	7.3	10.9	6.5	10.3	4.3	6.4	9.9	5.4	6.4	2.6	7.7	457

In Grade 7-2, 108 pupils who had repeated one or more grades were divided into two groups, the one being made up of 54 pupils who had repeated because of sickness, or transfer of school, etc., the other, of 54 pupils who had repeated because of failure to do the work of the grade. A control group of 54 pupils making normal progress was also used for the study.

TABLE LIII

AVERAGE ACCURACY IN ALL SETS. THREE AGE GROUPS. GRADE 8-2

	Accuracy
Young	86.3
Normal	84.3
Old	80.7

The average number of "rights" made in each set of the test by each of the groups is found in Table LIV, together with a state-

TABLE LIV

AVERAGE "RIGHTS" MADE IN EACH SET BY THREE PROMOTION GROUPS. GRADE 7-2. DATA FROM 162 PUPILS

Group	Set															Total Units
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	
Regular	30.2	22.1	19.2	20.8	7.4	9.8	6.0	9.0	3.7	7.2	8.5	4.2	4.9	1.5	5.4	400
Irregular	29.3	22.4	21.0	20.2	7.4	8.5	5.8	9.8	2.6	5.7	8.3	4.2	4.7	1.2	4.8	376
Failures	28.6	20.2	19.2	19.3	6.8	8.0	5.4	8.8	2.5	4.6	7.6	3.7	4.0	1.1	4.2	342

ment of total scores. An examination of the table shows quite considerable differences between the groups. The "irregular" pupils are superior to the "failures," and the "regular" pupils are superior to the "irregular." These differences are also more marked in examples of the more complex than in those of the simpler type. Table LV, the table of "attempts," shows differences of the same

TABLE LV

AVERAGE "ATTEMPTS" MADE IN EACH SET BY THREE PROMOTION GROUPS. GRADE 7-2. DATA FROM 162 PUPILS

Group	Set															Total Units
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	
Regular	30.3	22.4	20.4	21.1	7.7	10.2	6.6	11.7	4.5	8.4	9.0	5.6	6.1	2.3	8.6	472
Irregular	29.6	22.7	21.9	20.7	7.9	9.3	6.5	12.1	3.4	6.9	8.5	5.2	5.7	2.0	8.3	446
Failures	29.0	20.6	20.7	19.9	7.2	8.8	6.3	12.0	3.6	5.9	8.1	5.1	5.6	2.0	8.0	427

order. One difference, however, should be noted, and that is that the "irregular" pupils are more inferior to the "regular" pupils in "attempts" than in "rights." In other words, the difference between these two groups, as borne out by Table LVI, is not due

TABLE LVI

AVERAGE ACCURACY IN ALL SETS. THREE PROMOTION GROUPS. GRADE 7-2

	Accuracy
Regular	84.8
Irregular	84.3
Failures	80.1

to inaccuracy on the part of the "irregular" pupils, but rather to slowness or timidity. A graphical comparison of the three groups is found in Diagram 37.

Now the difference between the "failures" and the "irregular" pupils, which is in favor of the latter, is not at all surprising, but why should the "regular" pupils be superior to the "irregular" pupils? The latter have repeated the work of a grade because of sickness or transfer of schools and not because of inability to do the

work. In other words, they should not naturally be markedly different from the "regular" pupils, for any pupil may be sick and any pupil may be transferred from one school to another. Furthermore, it would seem that, if the repeating of a grade is a good thing for a pupil, the "irregular" pupil in a particular grade should be superior to the "regular" pupil of the same grade, since he has had

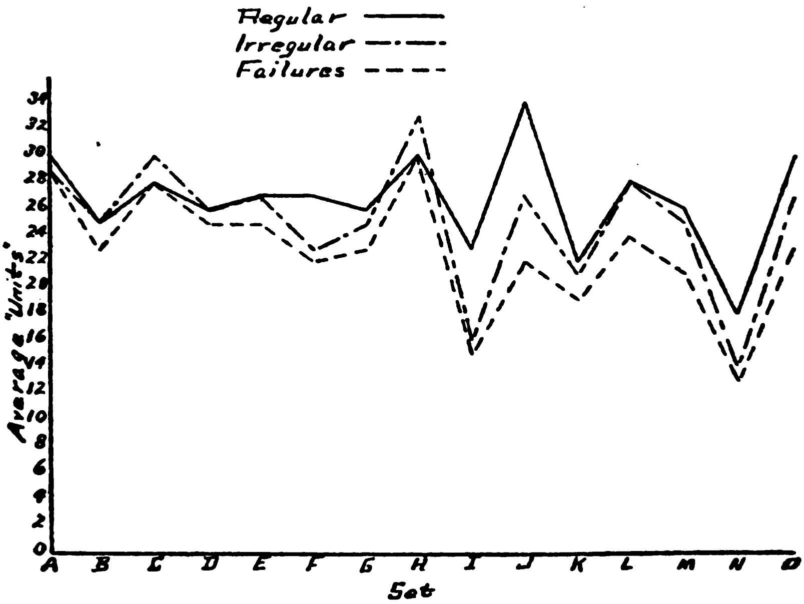


DIAGRAM 37.—Average "units" made in each set by three promotion groups in Grade 7-2.

a year's, or a portion of a year's, extra training. Since such is not the case, it must be that the repeating of the grade is not so valuable an experience to the pupil as some have been led to believe. From the evidence here set forth—inconclusive of course because of its paucity—the indications are that the repeating of a grade for whatever cause reacts upon the child and may even become a cause of failure.

The third study of promotion groups is based on the facts presented in Tables LVII, LVIII, and LIX. In the eighth grade 64 pupils who had repeated one or more grades were divided into two

groups, the one being composed of 32 pupils who had repeated below the sixth grade, the other, of 32 pupils who had repeated above the fifth grade.

TABLE LVII

AVERAGE "RIGHTS" MADE IN EACH SET BY TWO PROMOTION GROUPS. GRADE 8-2. DATA FROM 64 PUPILS

GROUP FAILING	SET															TOTAL UNITS
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	
Below sixth grade.....	30.8	23.4	20.1	21.7	7.4	10.8	5.8	8.1	3.9	5.7	10.7	4.6	4.9	1.7	4.8	403
Above fifth grade.....	30.0	23.4	21.5	23.1	7.6	9.9	5.9	7.8	3.3	5.6	9.2	4.0	5.3	1.5	4.3	387

The tables show slight differences in favor of the group failing below the sixth grade, but since the differences are so slight and the

TABLE LVIII

AVERAGE "ATTEMPTS" MADE IN EACH SET BY TWO PROMOTION GROUPS. GRADE 8-2. DATA FROM 64 PUPILS

GROUP FAILING	SET															TOTAL UNITS
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	
Below sixth grade....	31.3	23.9	22.8	22.3	8.2	11.9	7.1	11.4	5.0	7.4	11.0	6.1	6.8	2.6	7.5	493
Above fifth grade....	30.4	23.8	22.6	23.8	8.0	10.8	6.9	11.4	4.4	7.1	9.6	5.7	6.7	2.6	8.1	480

cases so few, whatever conclusions are drawn must be wholly tentative. Two explanations of the differences suggest them-

TABLE LIX

AVERAGE ACCURACY IN ALL SETS. TWO PROMOTION GROUPS. GRADE 8-2

	Accuracy
Failing below sixth grade........	81.7
Failing above fifth grade.........	80.6

selves: first, it may be that the pupil has more or less recovered from the failure in the earlier grades by the time he has reached

the eighth grade; second, it may be that failure in the lower grades is due to causes somewhat different from those that operate in the upper grades. In the former the pupil may be unable to realize what he is attending school for, and may therefore be quite indifferent to failure. That is, he may fail because of carelessness rather than because of inability. In the upper grades, on the other hand, failure may more often be the result of inability.

SUMMARY

1. With reference to the number of examples worked correctly by the pupils in the four age groups, it may be said that on the average the younger groups are superior to the older groups; that this superiority is more marked in the later than in the earlier grades; and that it is also more marked in the handling of the more complex than in the handling of the simpler types of examples.

2. In the number of examples attempted the study reveals no clear differences between any two of the four age groups.

3. On the average the younger pupils are found to be more accurate in their work than the older pupils; these differences are on the whole quite uniform from grade to grade; and they are more pronounced in the more complex than in the simpler examples.

4. A study of "fast," "regular," and "slow" pupils, as determined by promotion facts, reveals differences of the same order as those just stated concerning the age groups, the "fast" corresponding to the "young" and the "slow" to the "old."

5. A regrouping of these same pupils ("fast," "regular," and "slow") on the basis of age shows the differences to be more pronounced than when the pupils are grouped according to the rate of promotion.

6. This last statement would indicate a tendency to keep pupils in a grade because of youth.

7. "Failures" (pupils repeating because of inability to do the work of the grade) are inferior to "irregular" pupils (pupils repeating because of sickness, transfer of school, etc.), and the latter are inferior to "regular" pupils (pupils making just normal progress).

The relation between the two latter groups is of significance as indicating the injurious effects of repeating.

8. The evidence, inconclusive because of the small number of cases involved, indicates that among eighth-grade pupils the group made up of pupils who had failed below the sixth grade is superior to the group composed of pupils who had failed above the fifth grade.

CHAPTER VI

A COMPARISON OF THE ARITHMETICAL ABILITIES OF CERTAIN RACE GROUPS

The object of the present study is to make a comparison of the arithmetical abilities or attainments of the children of five races, viz., American, Hollander, German, Swede, and Slav, with a view (1) to determine whether or not there are racial differences, and (2) to discover the nature and extent of the differences, if such are found to exist.

METHOD

When the arithmetic test was given in the Grand Rapids schools, the principals of those schools in which several races were represented in the school population were requested to have the teachers indicate on the test sheet the race of each pupil taking the test in the upper grades. Mixed schools—that is, schools in which several races were represented—were chosen because it was thought that the comparison of races should be made on the basis of the records of individuals who had been subjected to the same school influences. It was considered that this would be a more valid method than the comparison of average records made by schools in which the various races were predominant, since such differences as would be found might very likely be due to differences in training.

The principals were told to call a pupil an "American" if both parents were born in this country. Otherwise the race of the pupil was to be indicated as that of the parents. Thus, if both parents were born in Holland the pupil was called a "Hollander"; if one was born in Holland and the other in Germany, the pupil was called "Hollander-German," and so on.

In answer to this request made of the principals, returns were secured from at least one of the upper grades in eleven schools. These records were examined and classified on the basis of race. If there was any doubt about the race of the pupil, the record was not used. For instance, if a pupil with a name like "Putowski"

gave his race as American, the record was discarded. The records made by pupils of mixed parentage were also rejected. The group "Slavs" is composed of Russians and Poles with a few Lithuanians and Bohemians; the "Swedes" include a few Norwegians and Danes; the other races are as the terms would indicate.

In Table LX appears a statement of the number of pupils of each of the five races whose records were finally selected for use in the study. The table also shows the distribution of the pupils of each race through the five upper grades, 6-2 to 8-2 inclusive, to which the study is confined. It will be noted that the Americans and Hollanders are well and about equally represented. The Slavs in Grades 8-1 and 8-2 and the Swedes in Grades 7-2 and 8-2 are especially poorly represented. These facts must be borne in mind when the results are interpreted.

TABLE LX

NUMBER OF PUPILS OF EACH RACE IN EACH GRADE WHOSE RECORDS WERE USED IN THIS STUDY

Race	Grade					Total
	6-2	7-1	7-2	8-1	8-2	
American..........	60	36	53	51	50	250
Hollander.........	47	49	55	56	50	257
German...........	25	31	23	17	21	117
Swede............	16	15	10	16	10	67
Slav..............	19	14	17	8	8	66
Total.........	167	145	158	148	139	757

The distribution of the pupils of these five races through the different schools and the composition of each class from which records were taken are presented in Table LXI. This table will be understood if read in this way: In the Coldbrook Elementary School the class of Grade 6-2 was composed of 24 pupils of whom 5 were clearly Americans, 2 Hollanders, 2 Germans, 1 a Swede, 4 Slavs, and 10 members of other, uncertain, or mixed races. The table is read in the same way for each of the grades and for each of the schools.

The method used in working up the data should now be explained in detail because of its complex character. If these five

TABLE LXI

NUMBER OF PUPILS OF EACH RACE IN EACH GRADE AND SCHOOL WHOSE RECORDS WERE USED IN THIS STUDY

School	Grade 6-2							Grade 7-1							Grade 7-2							Grade 8-1							Grade 8-2						
	Americans	Hollanders	Germans	Swedes	Slavs	Others	Total	Americans	Hollanders	Germans	Swedes	Slavs	Others	Total	Americans	Hollanders	Germans	Swedes	Slavs	Others	Total	Americans	Hollanders	Germans	Swedes	Slavs	Others	Total	Americans	Hollanders	Germans	Swedes	Slavs	Others	Total
Coldbrook	5	2	2	1	4	10	24	3	5	1	...	1	3	13	3	4	1	...	5	6	19	4	3	1	...	4	3	15	5	2	...	...	1	7	15
Diamond	1	10	...	...	1	9	21	6	9	1	2	...	12	30	3	6	...	...	1	3	13	10	11	2	...	...	22	45	4	9	3	2	...	12	30
East Leonard	2	6	...	...	...	13	21	1	5	...	...	2	8	16	5	4	1	1	1	7	19	3	5	1	...	...	3	12	13	10	...	1	...	9	33
Hall	4	4	1	...	...	17	26	8	10	6	2	...	13	39	4	9	5	...	...	7	25	3	8	2	1	...	6	20	...	...	...	...	...	...	...
Lexington	6	1	2	1	...	10	20	4	...	4	...	...	3	11	5	...	2	2	...	8	17	6	1	2	2	...	15	26	10	6	7	3	3	5	34
Michigan	13	2	3	2	1	12	33	3	3	3	2	1	7	19	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Oakdale	...	3	1	...	...	15	19	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Pine	4	2	3	5	1	4	19	1	2	7	2	2	1	15	4	3	2	3	1	2	15	...	...	...	...	...	...	...	...	...	...	...	...	...	...
South Division	9	4	2	1	...	16	32	1	1	3	2	...	21	28	6	...	6	...	...	8	20	3	8	1	1	...	11	24	5	5	1	1	...	4	16
Turner	6	11	6	...	7	16	46	5	6	2	...	5	13	31	7	5	...	1	1	7	21	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Union	10	2	5	6	5	6	34	4	8	4	5	3	8	32	16	24	6	3	8	22	79	22	20	8	12	4	38	104	13	18	10	3	4	22	70
Total	60	47	25	16	19	128	295	36	49	31	15	14	89	234	53	55	23	10	17	70	228	51	56	17	16	8	98	246	50	50	21	10	8	59	198

races had equal representation in each of the classes from which records have been taken, or if we had sufficient data so that equal numbers of the races might be taken from each class, as was done in the study of age groups, the question would be a very simple one. But, since the races are not equally represented in the classes and the data are strictly limited, some method must be found of eliminating differences of school training and methods of giving the test. Referring to Table LXI again, let us consider the problem as related to the records of the pupils in Grade 8-2, and for the moment let us confine ourselves to the records made by the Americans and the Hollanders. Five of the American pupils in this grade are taken from the Coldbrook School, while but 2 Hollanders are taken from the same school. Now if the training received in the Coldbrook is superior to that received in the other schools, and if the records made by the 50 American pupils and the records made by the 50 Hollanders in the grade be averaged without eliminating this factor of difference of training, the American group would be given an advantage because of its greater representation in a superior school.

The method adopted for eliminating differences in school training is as follows: The records made by the entire grade in one school are taken as a base, and a system of coefficients is computed by the use of which the records made in the other schools may be converted into the records of the school taken as a base. Thus the thing that is actually done is to determine what the records of the different pupils and groups of pupils would have been if they had all been in the same school, subjected to the same school influences. To be more concrete, let us again turn to Table LXI and indicate specifically how the method is applied in dealing with the race groups in Grade 8-2. The table shows that the data have been taken from Grade 8-2 of six schools. The next thing that must be done is to find the average records made by these six grades in each of the 15 sets of the test. These facts are presented in Table LXII. Now, taking the average score made in the Union School as a base, a coefficient is determined for each set in each school by dividing the record made in the Union School by the record made in each of the other schools. Thus a set of coefficients is found by which the records of the five schools may be converted into the records

made by the Union School, which means that differences in school training and differences in giving the test are overcome. By using the set of coefficients for any one of the schools, it is possible to

TABLE LXII

AVERAGE "RIGHTS" MADE IN EACH SET BY GRADE 8-2 IN SIX SCHOOLS

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Union	28.3	23.5	16.8	19.0	7.5	9.3	6.7	5.1	4.7	5.4	9.4	4.9	4.9	1.9	4.8
Coldbrook	28.8	28.8	22.3	27.2	8.3	13.5	7.9	8.8	5.5	6.8	12.2	6.5	6.8	3.1	5.3
Diamond	31.7	25.5	20.7	22.7	8.0	10.3	6.4	12.0	4.7	5.8	10.3	4.9	5.0	2.3	4.3
East Leonard	34.8	30.8	21.5	24.8	8.9	12.9	9.3	14.2	6.7	6.9	13.1	6.1	6.6	4.1	6.1
Lexington	32.0	30.8	23.5	24.6	9.0	12.3	7.7	7.8	4.7	6.7	11.0	5.8	6.5	2.7	6.0
South Division	32.5	26.5	20.5	23.0	7.6	10.0	6.3	8.5	4.0	5.0	10.3	4.3	5.3	1.8	4.0

determine what sort of a record a group of pupils in that school would have made if they had been trained and tested in the Union School. These coefficients are presented in Table LXIII.

TABLE LXIII

VALUE OF EACH EXAMPLE IN EACH SET FOR GRADE 8-2 IN EACH OF SIX SCHOOLS IN TERMS OF RECORD MADE BY GRADE 8-2 IN UNION

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Union	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Coldbrook	0.98	0.82	0.75	0.70	0.90	0.69	0.85	0.58	0.85	0.79	0.77	0.75	0.72	0.63	0.91
Diamond	0.89	0.92	0.81	0.84	0.94	0.90	1.05	0.43	1.00	0.93	0.91	1.00	0.98	0.83	1.12
East Leonard	0.81	0.76	0.78	0.77	0.84	0.72	0.72	0.36	0.70	0.78	0.72	0.80	0.74	0.46	0.79
Lexington	0.88	0.76	0.72	0.77	0.83	0.76	0.87	0.65	1.00	0.81	0.85	0.84	0.75	0.70	0.80
South Division	0.88	0.89	0.82	0.83	0.99	0.93	1.06	0.60	1.18	1.08	0.91	1.14	0.92	1.06	1.20

All that remains now is merely to point out the way in which these coefficients are applied to the race groups. Referring again to the section of Table LXI which presents the facts for Grade 8-2, we find 5 American pupils in the Coldbrook School. The total number of examples worked correctly in each of the sets by these 5 pupils is determined. Turning to Table LXIII we find the Coldbrook coefficient for Set A to be 0.98. The total number of "rights" made by the 5 pupils is multiplied by this quantity. The same thing is done for each of the other sets. Then we pass to the 4 pupils in the Diamond School and repeat the process, and like-

wise with the Americans in each of the other schools. Then the total scores thus made in each of the sets by the American pupils in the six schools are added. This grand total is then divided by 50, since that is the entire number of American pupils in the grade, and an average score is secured for each of the sets. Like procedure is followed for each of the other four race groups.

An objection which may be raised to this method is that in eliminating differences due to training and to the giving of the test race differences are also eliminated. The answer to this objection is that it would be valid if mixed schools—that is, schools in which several races are represented—were not used for the study. To the extent that a difference between the average records made by two schools is due to the presence of an inferior or a superior race group in one of the schools, the method does eliminate race differences as well as differences due to school training and methods of giving the test. But, since the schools are mixed, no one race can greatly affect the average score of a school. The objection thus resolves itself into the question of the racial composition of the classes from which the records used were taken. We must therefore refer again to Table LXI, in which the composition of each of these classes is given in detail. If it be remembered that the category of "others" includes representatives of races other than the five used in this study, pupils of mixed parentage, and pupils concerning whom there was some doubt as to race, the table shows that in no single instance does one race represent a majority of the class, and in only two cases, Diamond, Grades 6-2 and 7-2, does one approach representing a majority. Thus it is quite evident that the differences between any two schools cannot be attributed in any appreciable degree to the predominance of any race in one or the other of the schools. It may be that the method used in this study does minimize to some small extent racial differences, but to a very small extent if at all.

RESULTS

The average scores made in each set as computed by the method just outlined, by each of the five race groups in Grades 6-2 to 8-2, are found in Table LXIV. An examination of the table seems to

TABLE LXIV

AVERAGE "RIGHTS" IN EACH SET FOR EACH OF FIVE RACES IN GRADES 6-2 TO 8-2

	Grade 6-2					Grade 7-1					Grade 7-2					Grade 8-1					Grade 8-2				
	Americans	Hollanders	Germans	Swedes	Slavs	Americans	Hollanders	Germans	Swedes	Slavs	Americans	Hollanders	Germans	Swedes	Slavs	Americans	Hollanders	Germans	Swedes	Slavs	Americans	Hollanders	Germans	Swedes	Slavs
A......	26.6	27.2	28.7	28.1	31.5	24.4	25.6	27.1	24.8	27.0	28.5	29.7	31.1	27.7	32.8	29.2	29.4	31.0	30.6	33.9	27.4	26.8	29.4	30.8	29.6
B......	21.3	21.1	21.9	22.1	24.7	19.5	20.8	20.0	22.5	21.4	20.3	21.1	22.9	18.2	22.9	22.8	22.4	21.9	24.3	23.2	22.4	21.9	22.7	22.4	22.5
C......	17.5	17.2	17.7	17.8	19.4	20.7	19.6	19.0	20.6	20.1	17.6	16.1	18.5	16.1	18.9	20.4	17.9	19.5	19.4	22.9	16.8	16.3	16.8	16.9	17.4
D......	17.2	16.0	15.2	17.2	17.3	21.0	23.0	21.5	22.9	25.0	17.8	18.7	19.1	17.2	21.1	20.5	20.2	19.8	21.1	22.8	19.3	18.1	19.7	18.2	19.9
E......	6.4	6.2	6.0	6.4	7.2	7.1	17.5	7.0	7.5	7.3	6.1	6.2	7.0	5.5	6.8	7.5	7.7	8.2	7.7	7.8	6.7	6.7	7.1	7.3	7.1
F......	7.4	7.7	7.2	7.9	8.5	8.4	9.1	8.9	9.4	8.7	8.1	8.1	8.5	8.2	9.1	9.0	9.7	9.1	9.9	10.7	9.0	8.7	9.1	8.8	9.6
G......	4.4	5.0	5.6	6.1	5.9	5.2	6.1	5.8	6.2	6.6	5.3	5.6	5.9	5.3	6.1	5.7	5.9	6.1	5.9	6.7	6.3	6.1	6.7	6.5	5.5
H......	9.0	13.7	7.7	8.2	9.6	6.8	8.1	10.8	8.5	7.9	5.9	5.8	5.3	4.1	5.5	7.1	7.3	5.7	7.9	7.0	4.4	4.8	5.0	3.3	4.9
I......	2.9	3.2	3.0	3.8	3.3	2.6	3.7	2.8	2.8	2.6	2.9	2.8	3.0	3.3	3.1	3.5	3.8	3.6	4.3	4.3	4.3	4.2	4.0	4.7	3.6
J......	4.3	3.7	3.9	4.6	4.2	4.6	5.4	5.3	5.3	5.7	4.8	4.9	5.4	5.0	5.1	5.1	5.5	6.1	5.9	5.5	4.9	4.9	5.0	6.1	4.8
K......	8.6	7.7	7.0	7.6	7.8	6.4	6.4	5.7	7.1	7.4	8.9	8.3	8.9	8.7	9.6	9.0	9.5	9.3	10.5	11.1	9.0	8.1	8.3	9.4	8.9
L......	3.2	3.4	3.5	3.8	3.8	3.8	4.6	4.5	4.7	5.2	3.9	4.3	4.4	5.0	5.3	4.6	4.5	5.2	4.7	5.4	4.3	4.2	4.8	4.1	4.4
M......	4.2	4.3	4.8	4.4	4.9	4.7	5.5	4.6	4.9	5.3	4.1	4.3	5.2	4.7	5.1	5.2	5.0	5.3	5.5	5.9	4.4	4.5	4.8	5.2	5.3
N......	1.3	1.2	1.3	1.6	1.8	1.9	1.7	1.1	1.8	1.9	1.5	1.6	1.9	1.4	1.8	2.0	1.9	1.7	2.1	2.9	1.5	1.4	1.5	1.8	1.0
O......	5.1	5.0	3.8	5.8	5.6	7.4	8.2	7.3	7.7	6.1	3.2	3.2	3.9	3.1	3.2	5.4	5.2	5.2	5.7	5.3	5.0	5.3	4.4	4.0	5.1

TABLE LXV

AVERAGE NUMBER OF "UNITS" MADE IN ALL SETS, EXCEPT SETS H AND O (FRACTIONS), BY EACH OF FIVE RACES IN GRADES 6-2 TO 8-2

Race	Grade					Total	Average
	6-2	7-1	7-2	8-1	8-2		
American	279	301	295	336	320	1,531	306
Hollander	278	330	302	338	310	1,558	312
German	283	303	328	347	326	1,587	317
Swede	304	321	299	358	340	1,622	324
Slav	316	338	339	390	317	1,700	340

indicate that there are some differences and that they are in favor of the Swedes and Slavs. But since the differences do not appear to be consistently in any one set, Table LXV is presented, in which

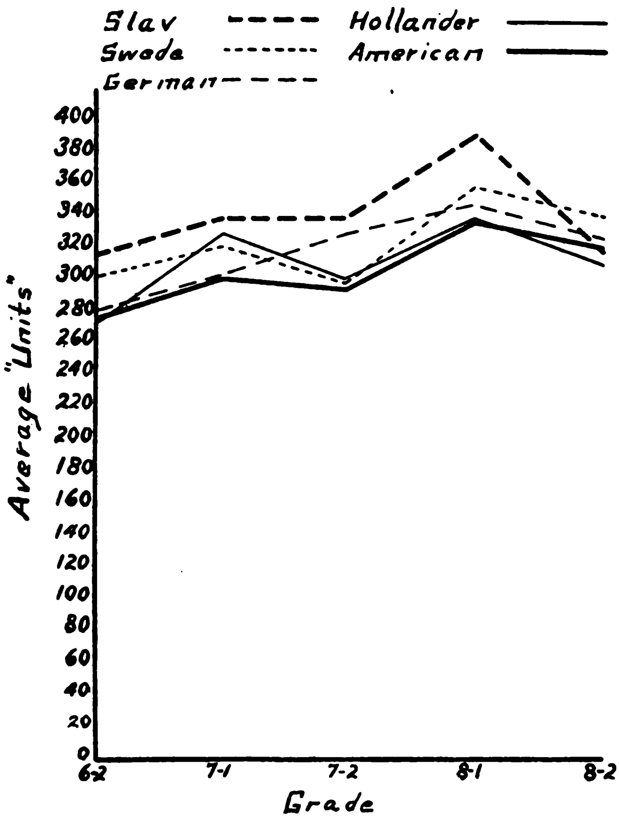


DIAGRAM 38.—A comparison of records made by five race groups

appear the total scores made by each of the races in the five grades. These total scores are obtained by applying the system of weights derived in chapter iii to the average scores given in Table LXIV.

Since these same facts are presented graphically in Diagrams 38 and 39, our attention may be turned toward them. The first of these diagrams shows at a glance that the American children are not prodigies when compared with the children of the other races. With the exception of the records for Grade 7-1 the Hollanders and Americans are in close agreement, and as compared with the other races they seem to be somewhat inferior. The Germans and the Swedes are intermediate groups, while the Slavs really seem to be superior. The curve for the Slavs becomes doubly significant when it is remembered that they had the smallest representation in Grades 8-1 and 8-2. It is in the latter grade only that this race does not hold first place.

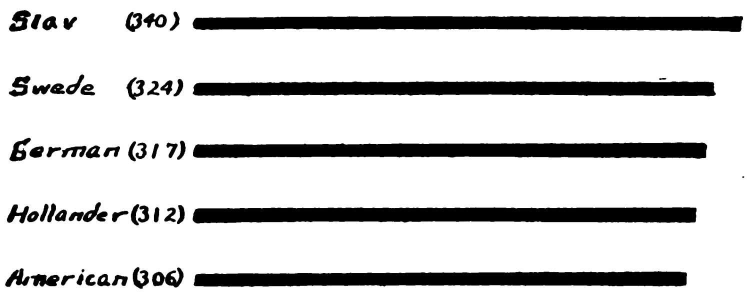


DIAGRAM 39.—A comparison of average numbers of units made in all sets (Sets H and O excluded) by five race groups—Grades 6-2 to 8-2 combined.

CONCLUSIONS

The conclusions that may be drawn from this study must of course be more or less tentative because of the small number of cases used. It seems safe, however, to conclude that the differences in arithmetical abilities of children of American parentage and children of Holland descent are very small, if they exist at all. The same may also be said of the German children, as compared with these two groups, while the indications are that the Swedes and the Slavs, and especially the latter, are superior. But as to whether these differences are due to the operation of biological or social factors, the present study furnishes no evidence.

CHAPTER VII

SUMMARY AND CONCLUSIONS

In this chapter a very brief summary of each of the studies will be made, together with a statement in each case of the conclusions that may be drawn. It should be reasserted, however, that these studies should all be supplemented by experimental evidence.

THE NATURE OF THE TEST AND COLLECTION OF DATA

1. The arithmetic test used in these studies was developed for the definite purpose of meeting a need felt by the staff of the Cleveland Survey, which was not met by any existing test.

2. The test as devised is a speed test which measures attainments and indicates weaknesses in the four fundamental operations and fractions.

3. In addition it tests knowledge of tables, the ability to add short columns, to bridge the "attention spans," and to "carry"; in subtraction it tests knowledge of the tables and the ability to "borrow"; in multiplication it tests knowledge of the tables, ability to "carry," and ability to add in connection with multiplication, as well as mastery of the mechanics of working the more complex examples in multiplication; in division it tests knowledge of the tables, ability to "carry" in short division, and ability to solve two types of examples in long division, the one involving neither "carrying" nor borrowing and the other involving both; and it tests the ability to apply these four fundamental operations to the working of examples in fractions.

4. The test was given to, and the results were secured from, 834 classes in the schools of Cleveland and Grand Rapids. In both cities the test was given almost entirely by the teachers. In Cleveland the teachers were inexperienced in giving tests, while in Grand Rapids they were all more or less familiar with the Courtis tests.

GENERAL RESULTS

1. Standard scores for the several sets in Grades 3–8 have been determined on the basis of results secured from Cleveland and

Grand Rapids pupils. A comparison of these scores with the Courtis standard scores in Sets A, B, C, and D indicates that the scores in these sets constitute rather accurate standards of attainment; and there seems to be no reason for believing that the scores in the other sets, with the possible exception of Set H, do not constitute equally accurate standards.

2. A system of weights has been derived whereby it is possible to equate the scores made in the several sets by an individual or group so that a single score may be secured to represent the general arithmetical attainment of the individual or group.

3. The use of the test is considered at some length. Methods of diagnosing individual, class, school, and city weaknesses are indicated.

4. Some very interesting facts are brought out in comparing grade distributions in the various types of examples: first, in the fundamentals the distribution-curve tends to become flattened with progress through the grades; second, the distribution-curve also tends to become flattened as we proceed from the less complex to the more complex types of examples in the fundamentals; third, as a general proposition in the fundamentals the distribution-curve representing the "rights" is flatter than that representing the "attempts"; fourth, in set O, fractions, the exact reverse of this last statement is true, the curve for the "attempts" being flatter than that for the "rights."

5. Tentative standards of accuracy for each of the sets in Grades 3–8 inclusive have been determined on the basis of results from Cleveland and Grand Rapids children.

6. Curves representing progress in accuracy through the grades and curves representing progress in the average number of examples worked are compared. The accuracy-curve takes the form of the learning-curve, while the "rights"-curve does not.

TYPES OF ERRORS

1. In the addition of the simple combinations the general proposition seems to be established that on the average those combinations whose sums exceed ten are more difficult than those whose sums are less than ten. To this general statement there are indi-

vidual exceptions which indicate the formation of peculiarly strong associations, some being right and others wrong. These peculiar associations vary among different groups. This would indicate that the formation of the association is to be accounted for in terms of the experience of the group rather than in the character of the combination itself.

2. In the simple subtraction combinations "bridging the tens" is found to be a relatively much more difficult operation than in the addition combinations. Freakish errors, on the other hand, are found to be less frequent in the former than in the latter. The understanding of the meaning of zero seems to accompany maturing of the pupil. This is indicated by a relatively large percentage of errors made on the combination 1—0 by fifth-grade pupils, whereas this combination presented but little difficulty to pupils in the eighth grade.

3. Practically all the errors made in the simple multiplication combinations are made in those combinations in which zero enters as one of the terms. Furthermore, it is a more difficult mental operation to multiply a quantity by zero than to perform the reverse operation, multiply zero by the quantity. And a pupil may have difficulty with the zero in the simple combinations, yet be quite able to handle it in the more complex examples, and vice versa. In the complex multiplication examples the most frequent error is made in multiplying.

4. In the simple division combinations the most frequent error is made in dividing a quantity by itself. The result given is zero, showing a confusion between the division and subtraction processes. In long division the demand for multiplication accounts for most of the errors.

5. The typical errors made in working fractions indicate, as a general rule, a slavish adherence to the mechanics of fractions and show emphasis upon method rather than upon an understanding of the process. There consequently follows a great deal of confusion of methods on the part of the pupil.

6. In the addition and subtraction of fractions of like denominator there is a tendency to add both numerators and denominators in the one case and subtract them in the other.

Monograph No. 5 (In Press)

Types of Reading Ability as Exhibited through Tests and Laboratory Experiments. By CLARENCE TRUMAN GRAY, PH.D., Instructor in Education, University of Texas.

Mr. Gray carried on a series of studies of reading for one year with the aid of a subsidy supplied by the General Education Board. He photographed the eyes of a number of children who had been selected through carefully conducted tests in the Elementary School and in the High School of the University of Chicago. By means of the tests he was able to distinguish different types of ability to read; by means of the laboratory experiments he arrived at an explanation of a number of these types. The work undertaken by Mr. Gray was carried further by a number of teachers who worked on these special cases and improved the ability of the pupils through special training. Reports on these special cases are included in the monograph.

Monograph No. 6 (In Press)

Survey of the Kindergartens of Richmond, Indiana. By ALICE TEMPLE, ED.B. About 60 pages.

The school system of Richmond, Indiana, has kindergartens in all its buildings. The Board, recognizing the demand for a closer co-ordination of this division of the schools with the grades, arranged for a survey. Professor Alice Temple, head of the Kindergarten Department of the School of Education of the University of Chicago, made the survey. It is a systematic study of the course, the equipment, the training of the teachers, and the possibility of articulation with the first grade. It is the first complete kindergarten survey.
